EE8501 POWER SYSTEM ANALYSIS

OBJECTIVES:

 To model the power system under steady state operating condition
 To understand and apply iterative techniques for power flow analysis
 To model and carry out short circuit studies on power system
 To model and analyze stability problems in power system

UNIT I POWER SYSTEM 9

Need for system planning and operational studies - Power scenario in India - Power system components – Representation - Single line diagram - per unit quantities - p.u. impedance diagram - p.u. reactance diagram - Network graph, Bus incidence matrix, Primitive parameters, Bus admittance matrix from primitive parameters - Representation of off- nominal transformer - Formation of bus admittance matrix of large power network.

UNIT II POWER FLOW ANALYSIS 9

Bus classification - Formulation of Power Flow problem in polar coordinates - Power flow solution using Gauss Seidel method - Handling of Voltage controlled buses - Power Flow Solution by Newton Raphson method.

UNIT III SYMMETRICAL FAULT ANALYSIS 9

Assumptions in short circuit analysis - Symmetrical short circuit analysis using Thevenin's theorem - Bus Impedance matrix building algorithm (without mutual coupling) – Symmetrical fault analysis through bus impedance matrix - Post fault bus voltages - Fault level – Current limiting reactors.

UNIT IV UNSYMMETRICAL FAULT ANALYSIS 9

Symmetrical components - Sequence impedances - Sequence networks - Analysis of unsymmetrical faults at generator terminals: LG, LL and LLG - unsymmetrical fault occurring at any point in a power system - computation of post fault currents in symmetrical component and phasor domains.

UNIT V STABILITY ANALYSIS 9

Classification of power system stability – Rotor angle stability - Swing equation – Swing curve - Power-Angle equation - Equal area criterion - Critical clearing angle and time - Classical step-by-step solution of the swing equation – modified Euler method.

TOTAL : 45 PERIODS

OUTCOMES:

- Ability to model the power system under steady state operating condition
- Ability to understand and apply iterative techniques for power flow analysis
- Ability to model and carry out short circuit studies on power system
- Ability to model and analyze stability problems in power system
- Ability to acquire knowledge on Fault analysis.
- Ability to model and understand various power system components and carry out power flow, short circuit and stability studies.

TEXT BOOKS:

1. John J. Grainger, William D. Stevenson, Jr, 'Power System Analysis', Mc Graw Hill Education (India) Private Limited, New Delhi, 2015.
2. Kothari D.P. and Nagrath I.J., 'Power System Engineering', Tata McGraw-Hill Education, Second Edition, 2008.
3. Hadi Saadat, 'Power System Analysis', Tata McGraw Hill Education Pvt. Ltd., New Delhi, 21st reprint, 2010.

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1. Pai M A, 'Computer Techniques in Power System Analysis', Tata Mc Graw-Hill Publishing Company Ltd., New Delhi, Second Edition, 2007.
J. Duncan Glover, Mulukutla S.Sarma, Thomas J. Overbye, 'Power System Analysis & Design', Cengage Learning, Fifth Edition, 2012.
3. Gupta B.R., 'Power System - Analysis and Design', S. Chand Publishing, 2001.
4. Kundur P., 'Power System Stability and Control', Tata McGraw Hill Education Pvt. Ltd., New Delhi, 10th reprint, 2010.

Modern Power System :

The power system network of today is a complex interconnected network.

A power system can be divided into 4 major parts.

1. Generation.
2. Transmission and Subtransmission.
3. Distribution.
4. Loads.

Generation :

Several Components are used for generation of Electricity like Synchronous generators, prime mover, steam turbines, Hydraulic turbines, transformers etc..

Synchronous Generators : Synchronous Generators or alternator have two synchronously rotating fields. one field is produced by the rotor driven at synchronous speed and excited by dc current. The other field is produced in the stator windings by 3-ϕ armature currents.

* The dc current for the rotor windings is provided by excitation systems. In older units, the exciters are dc generators mounted on the same shaft, providing excitation through slip rings. Today's systems use ac generators with rotating rectifiers, known as brushless excitation

systems. The generator excitation system maintains generat[or]
Voltage and controls the reactive power flow.

* In a power plant, the size of generators can vary from 50MW to 1500MW.

Prime mover:

* Prime mover is the source of mechanical power.

* It may be hydraulic turbines at waterfalls (or) steam turbines whose energy comes from burning of coal, gas, nuclear fuel (or) gas turbines.

* Steam turbines operates at relatively high speeds of 3600 or 1800 rpm.

* The generators to which they are coupled are cylindrical rotor, 2-pole for 3600rpm or 4 pole for 1800rp[m]

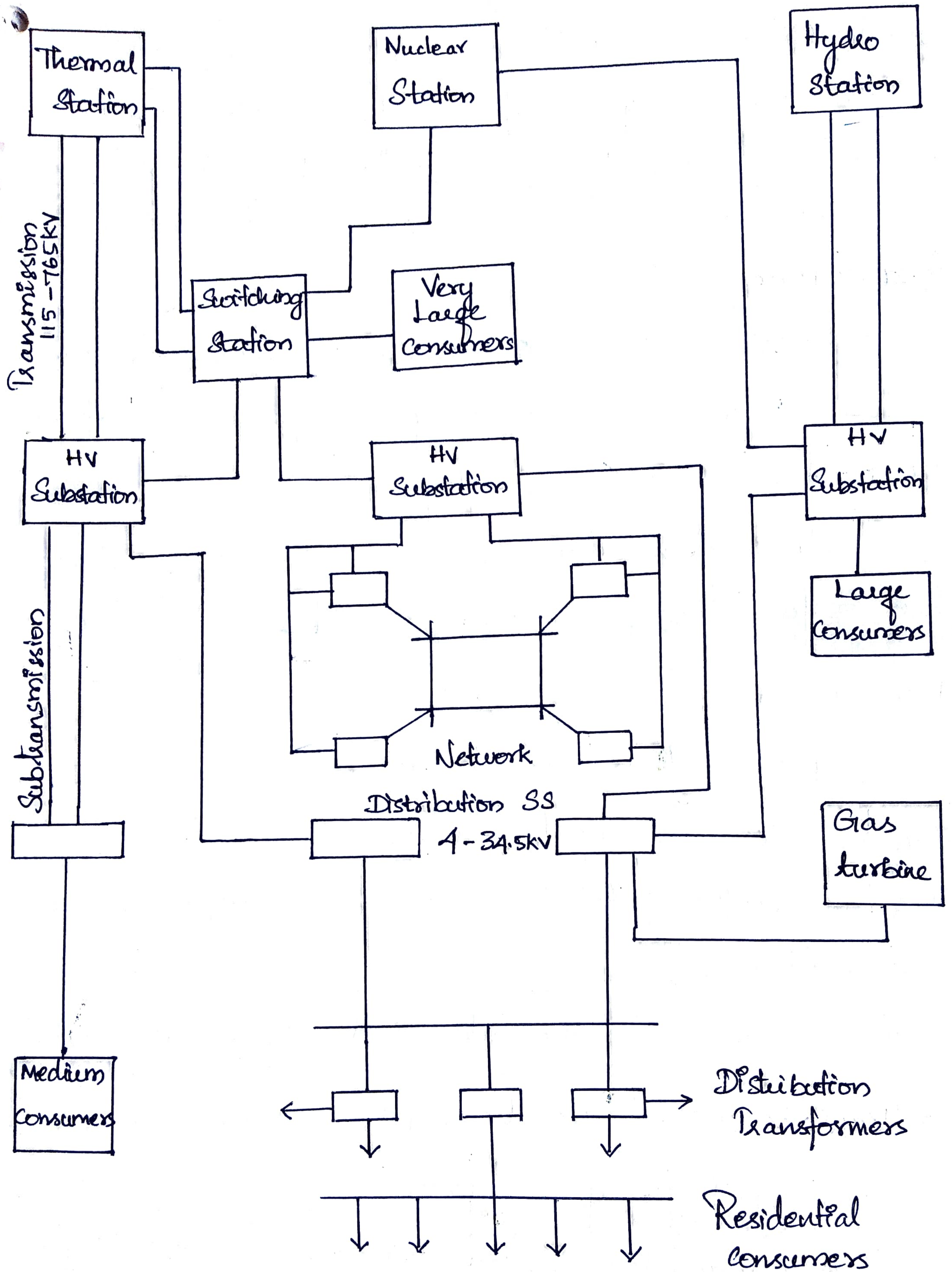
* In power station several generators are operated in parallel in the power grid to provide the total power needed. They are connected in a common point called a bus.

Transformers:

* The major component of power system is transform[er]

* It transfers power with very high efficiency from one level of voltage to another level.

* The power ...



BASIC COMPONENTS OF A POWER SYSTEM

the same as the primary except for losses in the transformer.

* Using a step-up transformer of turns ratio α will reduce the secondary current by a ratio of 1/α. This will reduce losses in the line, which makes the transmission of power over long distances possible.

* Step-up transformers are used for transmission of power at the sending end side and again step down transformers are used at the receiving side.

* In modern utility system, the power may undergo four or five transformations from generator to ultimate user.

Transmission

* The purpose of an overhead transmission network is to transfer electric energy from generating units at various locations to the distribution system which ultimately supplies the load.

* Transmission lines also interconnect neighbouring utilities.

* Transmission line are also useful in transfer of power between regions during emergencies.

* High voltage transmission lines are terminated in substations which are called high voltage substations, receiving substations, or primary substations.

* Very large industrial consumers ... transmission system.

* The portion of the transmission system that connects the high voltage substations through step down transformers to the distribution substations are called the subtransmission network.

* Typically, the subtransmission voltage level ranges from 69 to 138 KV.

* Some large industrial customers may be served from the subtransmission system.

Distribution:

* The distribution system is that part which connects the distribution substations to consumers' equipment.

* The primary distribution lines are usually in the range of 4 to 34.5 KV and supply the load.

* Some small industrial customers are served directly by the primary feeders.

* The secondary distribution network reduces the voltage for utilisation by commercial and residential consumers.

* Distribution systems are both overhead and underground.

* The secondary distribution serves most of the customers ... levels of 240/120V, single phase, three-wire, 208/120V, ... phase, 4 wire.

Loads:

* Loads of power systems are divided into industrial, commercial and residential.

* Very large industrial loads may be served from the transmission system.

* Large industrial loads are served directly from the subtransmission network, and small industrial loads are served from the primary distribution network.

* Most of the industrial loads are induction motors. These loads are functions of voltage and frequency.

* Commercial and residential loads consists of lighting heating and cooling.

* The magnitude of load varies throughout the day.

* The greatest value of load during a 24-hour period is called peak or maximum demand.

* Small peaking generators may be used to meet the peak load that occurs only for a few hours.

Load factor:-

* The load factor is the ratio of "average load" over a designated period of time to the "peak load" occurring in that period.

* Inorder to calculate the usefulness of generating plant, load factor is calculated.

* Load factor may be given for a day, ...

$$\text{Daily Load factor} = \frac{\text{average load} \times 24\,hr}{\text{Peak load} \times 24\,hr}$$

$$\text{Annual Load factor} = \frac{\text{total annual energy}}{\text{Peak load} \times 8760\,hr}$$

Utilization Factor :

It is the ratio of maximum demand to the installed capacity.

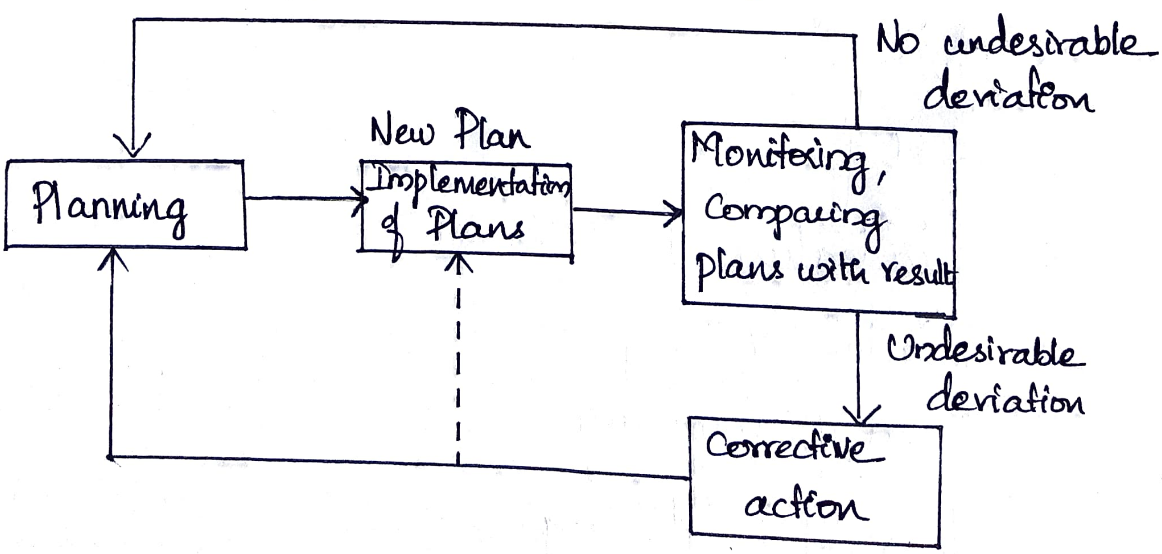
Plant Factor :

It is the ratio of annual energy generation to the plant capacity × 8760 hr.

* These factors indicate how well the system capacity is utilised and operated.

Analysis for System Planning and Operational Studies

The need for planning and operational analysis is explained in the block diagram given below.



analysis or studies are done, They are.

* Load flow studies.
* Short circuit or fault studies.
* Stability studies.

Load flow studies:

Load flow study is the determination of Voltage, current, real and reactive power at various points in an electric network.

The main objective of load flow studies is to identify the potential problems in terms of unacceptable Voltage conditions, overloading, decreasing reliability or any failure of the transmission system to meet the performance criteria.

These studies also provide information about,

1. Frequency.
2. Bus Bar Voltage profile.
3. Effect of inphase.
4. quadrature Boost Voltages.
5. Reactive voltamperes.
6. Effect of temporary loss of generation + transmission.
7. Power flow through transmission line.
8. Transformer load + other elements of power s/m

for a specified load demand.

9. Net interchange of power with adjoining power system. etc..

* Alternative plans for future expansion to meet new load demands can be analyzed.

* To keep pace with increasing load growth, new generating facilities, new lines, new interconnections or extensions are planned and added periodically.

* Poor voltages existing in any parts of the system can be detected.

Short circuit (or) Fault (Studies) Calculations:

A fault in a circuit is any failure which interferes with the normal flow of current. Fault calculations include determining the currents for various types of faults at various locations in the system.

The main objective of short circuit studies are.

1. To determine the current interrupting capacity of the circuit Breakers so that the faulted equipment can be isolated successfully.
2. to calculate the voltages during faulted conditions that affect insulation co-ordination.
3. To design grounding systems.
4. To design lightning arrester systems.

The selection of circuit Breakers depends on the current flowing immediately after the fault occurs and the current which the breaker must interrupt.

Some types of faults include line to ground fault, line to line fault, double line to ground fault, three phase to ground fault.

The data collected by short circuit studies is used to select fuses, circuit Breakers, relays etc. to protect the system from abnormal condition.

<u>Stability Studies</u> :

Stability studies are performed in order to ensure that the system remains stable following (after) a severe fault or disturbance.

Power system stability is a condition in which the various synchronous machines of the system remain in synchronism with each other.

There are 3 types of stability.

i) Steady state stability.
ii) Transient stability.
iii) Dynamic stability.

The steady state stability is defined as the ability of the power system to remain in synchronism, after a slow load change.

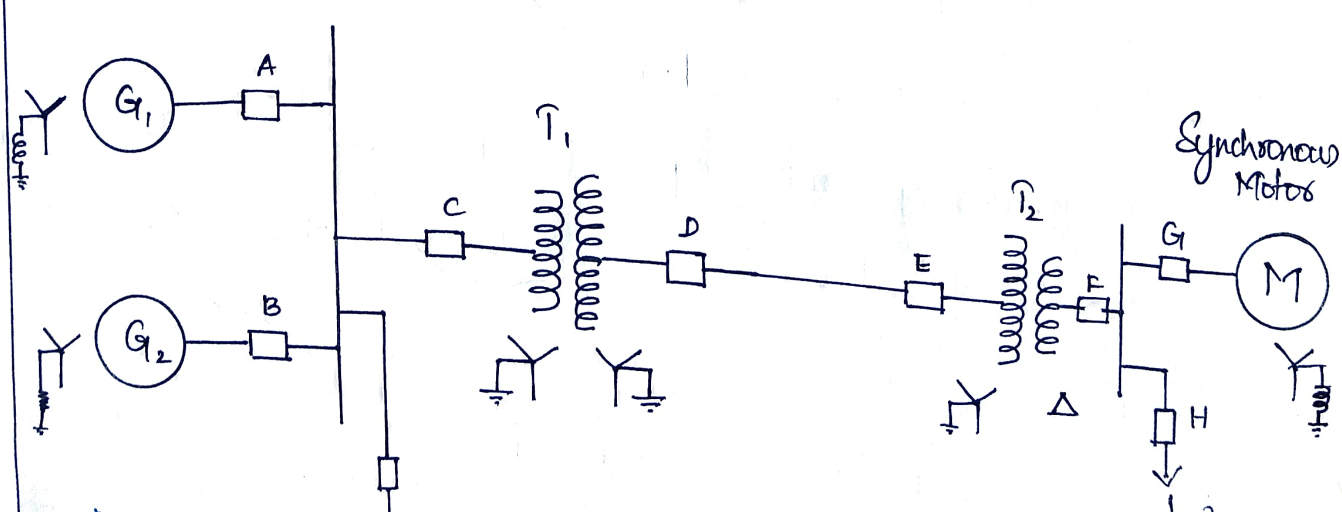
Transient stability is the ability of the power system to remain in synchronism after large sudden disturbance caused by faults and switching operations.

The ability of a power system to maintain stability under continuous small disturbances is termed as Dynamic stability.

These studies are helpful in determining the following.

1. Changes in bus voltage during disturbances.
2. Power flow during disturbance.
3. Nature of relaying system needed.
4. Power transfer capacity.
5. Critical clearing time etc.

Single Line Diagram:



* A power system consists of number of Generators, Transformers, Transmission lines, loads etc.

* These equipments are shown by standard symbols in

* Any particular component may or may not be shown depending on the type of system study. For eg. circuit Breakers need not be shown in a load flow study but must for protection system.

* Two generators, one grounded through reactor and one grounded through resistor is shown.

* The two generators are connected to bus and then to transmission line through transformer.

* The transformer T_1 with both primary and secondary neutrals are solidly earthed is shown.

* The transformer T_2 with secondary neutral solidly earthed and the primary is delta connected.

* Loads L_1 and L_2 and circuit Breakers (A, B, C... H) are shown as small box.

* Synchronous motor grounded through a reactor is connected to bus to the opposite end of the transmission line.

<u>Impedance Diagram:</u>

* The impedance diagram of a power system is the equivalent circuit of the power system in which various components of the power system are represented by their simplified equivalent circuits.

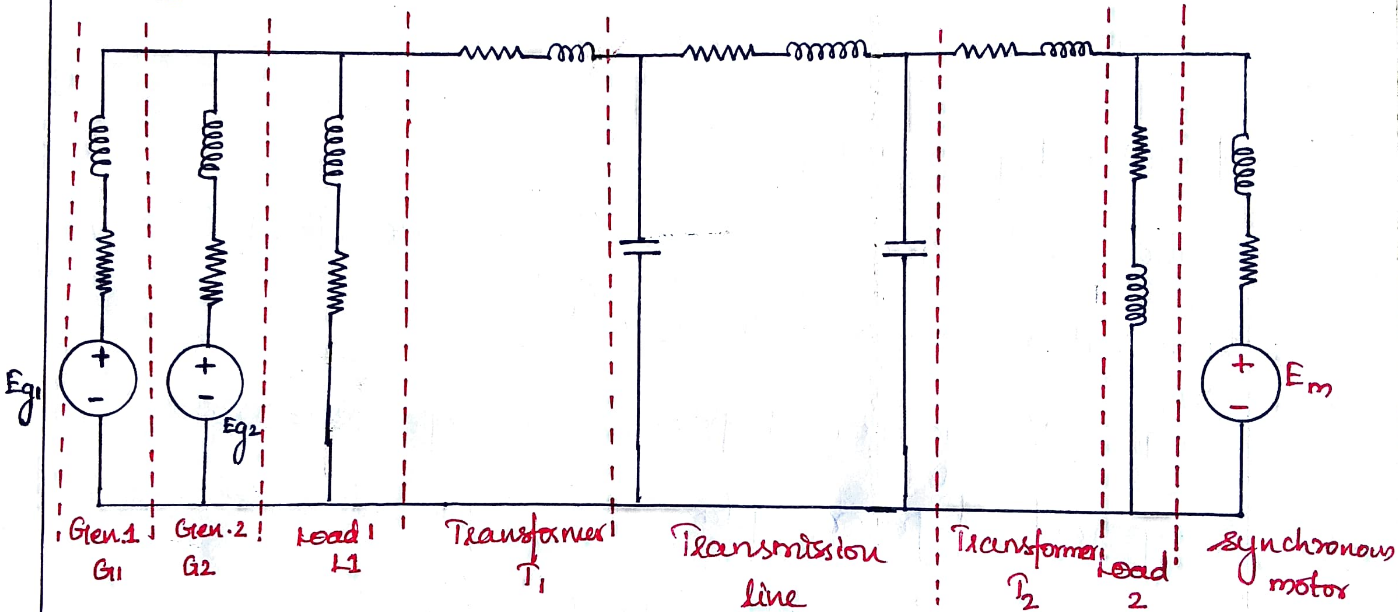
* This impedance diagram is used to analyse the

performance of a system under load conditions i.e. for load flow studies and fault studies.

* While drawing impedance diagram, the impedance connected b/w generator neutral and ground is neglected.

* No-load equivalent circuit of transformer is neglected.

* If inductive reactance is very high, the resistance can be omitted.

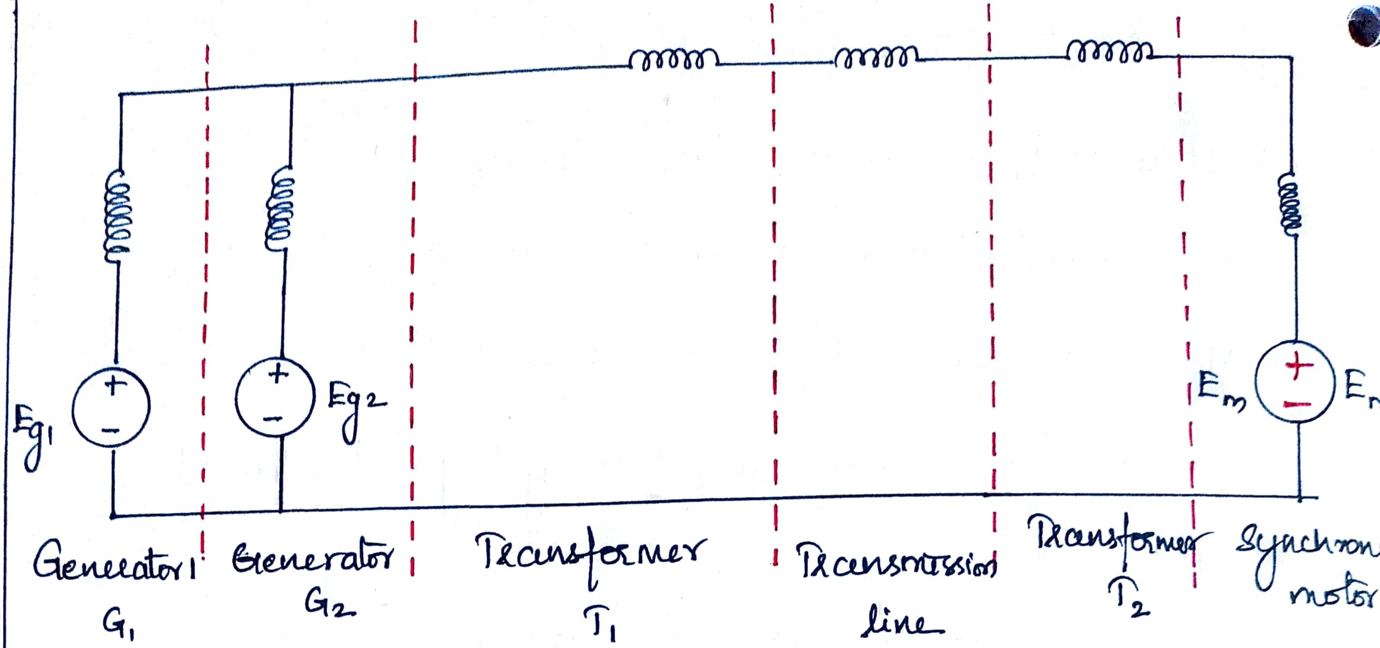


Reactance Diagram:

⇒ The reactance diagram can be obtained from impedance diagram if we omit the following.

* static loads.
* all Resistances.
* No load equivalent circuit.
* capacitance of transmission line

⇒ The reactance diagram is used for fault calculations.



Per Unit Representation:

The electric power transmission lines are operated at very high voltage levels and power levels.

The voltage, current, power and impedance ratings of components of power system are expressed with reference to a common value called 'base value'.

Then all the voltage, power, current and impedance ratings of the components are expressed as a percent of per unit of the base value.

Definition:

The per unit value of any quantity is defined as the ratio of the actual value of base quantity to base value.

$$\text{Per unit value} = \frac{\text{Actual Value}}{\text{Base Value}}$$

Formula:

Base current, $I_b = \frac{KVA_b}{KV_b}$ (amps) —— ①

Base impedance, $Z_b = \frac{KV_b \times 1000}{I_b}$ (Ω) —— ②.

subs. eq^n. ① in ②.

$$Z_b = \frac{KV_b \times 1000}{KVA_b / KV_b}$$

$$= \frac{(KV_b)^2}{KVA_b / 1000}$$

$$\boxed{Z_b = \frac{(KV_b)^2}{MVA_b}}$$

$$\text{Per Unit impedance} = \frac{\text{Actual impedance}, \Omega}{\text{Base impedance}, \Omega}.$$

Changing Base of per unit Quantities:

$$Z_{pu,new} = Z_{pu,old} \times \left[\frac{KV_{b,old}}{KV_{b,new}}\right]^2 \times \left[\frac{MVA_{b,new}}{MVA_{b,old}}\right]$$

Problems:

1. A three phase generator with rating 1000KVA, 33KV has its armature resistance and synchronous reactance as 20Ω/phase and 70Ω/phase. Calculate p.u. impedance of the generator.

Given:

$KV_b = 33KV$.

$KVA_b = 1000KVA$.

Formula:

$$\text{p.u impedance}, Z_{pu} = \frac{\text{Actual impedance}}{\text{Base impedance}}.$$

$$\text{Base impedance}, Z_b = \frac{(KV_b)^2}{MVA_b}$$

Solution:

(i) $$\text{Base impedance}, Z_b = \frac{(KV_b)^2}{MVA_b}$$

$$= \frac{(33)^2}{1}$$

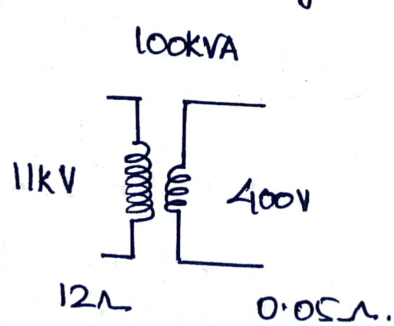
$$= 1089\,\Omega$$

$$MVA_b = \frac{KVA_b}{1000} = \frac{1000KVA}{1000} = 1\,MVA$$

(ii) $$\text{P.u impedance } Z_{pu} = \frac{20+j70}{1089}$$

$$= 0.018 + j0.064 \text{ p.u.}$$

2. A, 3ϕ, Δ-Y transformer with rating 100KVA, 11kV/400V has its primary and secondary leakage reactance as 12Ω/phase and 0.05Ω/phase respectively. Calculate p.u reactance of transformer.



Given:-

Base chosen: "primary"

$KV_b = 11kV$; ...

Formula Used:

(i) $X_{pu} = \dfrac{\text{Leakage reactance}}{\text{Base impedance}}$.

(ii) Base impedance, $Z_b = \dfrac{(KV_b)^2}{MVA_b}$

(iii) Leakage Reactance, $X_{01} = X_1 + X_2' = X_1 + X_2/K^2$.

Solution:

(i) Base impedance, $Z_b = \dfrac{(11)^2}{0.1}$

$$\boxed{Z_b = 1210\,\Omega}$$

(ii) Leakage Reactance, $X_{01} = X_1 + X_2/K^2$

$= 12 + \dfrac{0.05}{(0.036)^2}$

$= 50.5\ \Omega/\text{phase}$

$K = \dfrac{E_2}{E_1}$

$= \dfrac{400}{11,000}$

$= 0.036$.

(iii) $X_{pu} = \dfrac{50.5}{1210}$

$= 0.041$ p.u.

Procedure to draw Reactance Diagram:

Step: 1 : Select KV_b, KVA_b

Step: 2 : Transformer Rating changes: The Various sections of power system works at different Voltage levels because of transformers. Hence KV_b of one section of txfr. should be converted to KV_b corresponding to another section based on K.

kV_b on LT section = kV_b on HT section × $\frac{\text{LT Voltage rating}}{\text{HT Voltage rating}}$

kV_b on HT section = kV_b on LT section × HT/LT.

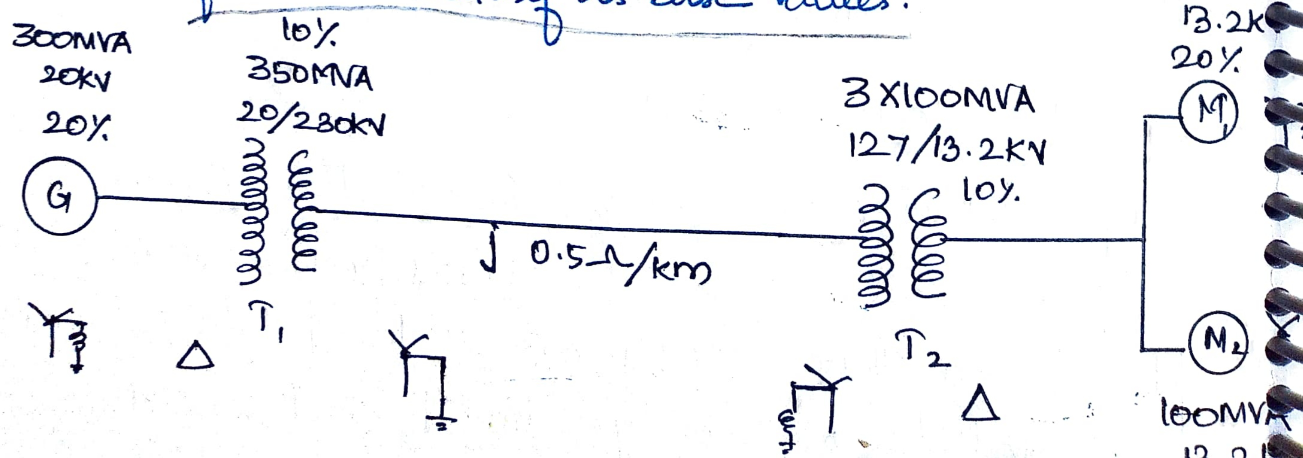
Step : 3 : when specified reactance of the component is in ohms then,

$$\text{P.u reactance} = \frac{\text{Actual reactance in } \Omega s}{\text{Base impedance}}$$

* when specified reactance is in p.u, consider it as old base values & selected base values as new bases. Then calculate p.u using,

$$X_{p.u,new} = X_{p.u,old} \times \left[\frac{kV_{b,old}}{kV_{b,new}}\right]^2 \times \left[\frac{MVA_{b,new}}{MVA_{b,old}}\right]$$

3. A 300MVA, 20kV, 3ϕ generator has a subtransient reactance of 20%. The generator supplies 2 synchronous motors through a 64km transmission line having transformers at both ends as shown. In this T_1 is a 3ϕ transformer and T_2 is made of 3 single phase transformer of rating 100MVA, 127/13.2kV, 10% reactance. Series reactance of transmission line is 0.5Ω/km. Draw the reactance diagram with all the reactances marked in p.u. Select the generator rating as base values.



Generator rating:

$KV_{b,new} = 20KV$ 1)

$MVA_{b,new} = 300MVA$ 2)

KVb — speed breaker xfr

Reactance of Generator G:

p.u reactance = 0.2 pu.

Reactance of Transformer T_1:

New p.u reactance of transformer, T_1

$$= X_{pu,old}\left[\frac{KV_{b,old}}{KV_{b,new}}\right]^2 \times \left[\frac{MVA_{b,new}}{MVA_{b,old}}\right]$$

$$= 0.1 \times \left[\frac{20}{20}\right]^2 \times \frac{300}{350}$$

$$= 0.0857 \text{ p.u.}$$

Reactance of Transmission line:

Total reactance $= 0.5\Omega/km \times 64 km.$

$= 32\Omega.$

Base KV on HT side of transformer T_1 = Base KV on LT side × $\frac{\text{HT Vge rating}}{\text{LT Vge rating}}$

$$= 20 \times \frac{230}{20}$$

$$= 230kV.$$

Base impedance, $Z_b = \frac{(KV_b)^2}{MVA_b} = \frac{(230)^2}{300} = 176.3\Omega.$

Per unit reactance of transmission line $=\dfrac{\text{Actual reactance in Ohms}}{\text{Base impedance}}$

$$=\frac{32}{176.33}$$

$$=0.18147\,\Omega.$$

Reactance of Transformer T_2 :

Base kV on LT side of Transformer T_2 $=$ Base kV on HT side $\times \dfrac{\text{LT Voltage}}{\text{HT Voltage}}$

$$=230\times\frac{13.2}{127\times\sqrt{3}}$$

$$=13.80\,kV.$$

New p.u reactance of Transformer T_2 $=X_{pu,old}\left[\dfrac{KV_{b,old}}{KV_{b,new}}\right]^2\times\left[\dfrac{MVA_{b,new}}{MVA_{b,old}}\right]$

$$=0.1\times\left[\frac{13.2}{13.8}\right]^2\times\left[\frac{300}{8\times100}\right]$$

$$=0.0915\text{ p.u.}$$

Reactance of Motor M_1 :

New p.u reactance of Motor, M_1 $=X_{pu,old}\times\left[\dfrac{KV_{b,old}}{KV_{b,new}}\right]^2\times\left[\dfrac{MVA_{b,new}}{MVA_{b,old}}\right]$

$$=0.2\times\left[\frac{13.2}{13.8}\right]^2\times\left[\frac{300}{200}\right]$$

$$=0.2744\text{ p.u}$$

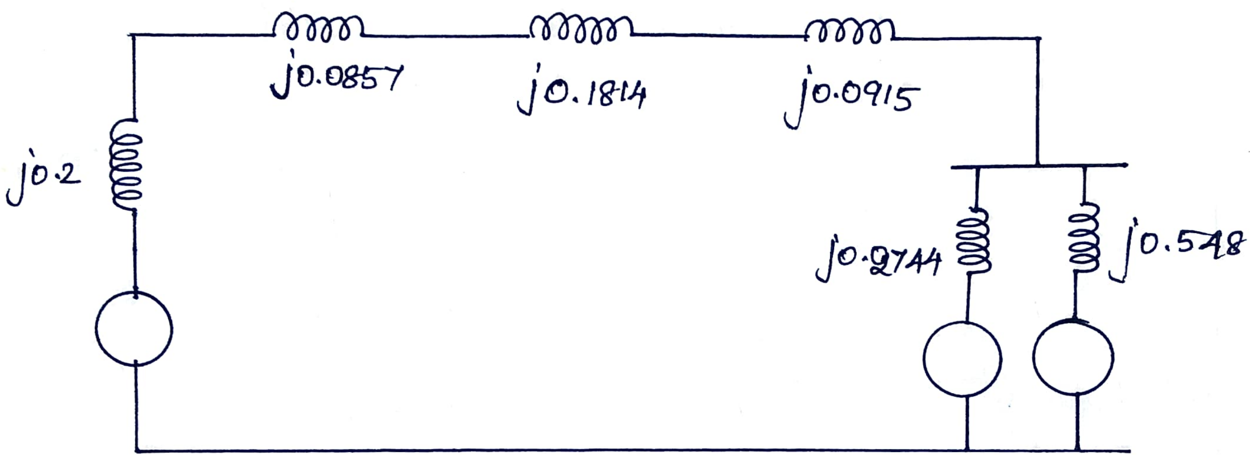
Reactance of Motor, M_2 :

$$\text{New p.u reactance} = X_{p.u,old}\left[\frac{KV_{b,old}}{KV_{b,new}}\right]^2 \times \left[\frac{MVA_{new}}{MVA_{b,old}}\right]$$

$$= 0.2 \times \left[\frac{13.2}{13.8}\right]^2 \times \left[\frac{300}{100}\right]$$

$$= 0.548 \text{ p.u.}$$

Reactance Diagram :



2. A 120 MVA, 19.5 KV generator has a synchronous reactance of 0.15 p.u. and it is connected to a transmission line through a transformer rated 150 MVA, 230/18 KV (Y/Δ) with X = 0.1 p.u.

(a) calculate the p.u reactances by taking generator rating as base values.

(b) calculate the p.u reactance by taking transformer rating as base values.

(c) calculate the p.u reactance for a base value of 100 MVA and 220 KV on HT side of transformer.

(replace HT side vge as 220kv)

(a) Generator rating as base values:

$KV_{b,new} = 19.5 KV.$

$MVA_{b,new} = 120 MVA.$

Same KV and MVA ratings.

so generator reactance = 0.15 p.u.

p.u reactance of transformer $= X_{pu,old} \times \left[\frac{KV_{b,old}}{KV_{b,new}}\right]^2 \times \left[\frac{MVA_{b,new}}{MVA_{b,old}}\right]$

$= 0.1 \times \left[\frac{18}{19.5}\right]^2 \times \left[\frac{120}{150}\right]$

$= 0.0681$ p.u.

(b) Transformer rating as base values:

$MVA_{b,new} = 150 MVA.$

$KV_{b,new} = 18 KV.$

For transformer KV and MVA are same, so the p.u value is also the same,

Transformer reactance = 0.1 pu.

Generator reactance $= X_{pu,old} \times \left[\frac{KV_{b,old}}{KV_{b,new}}\right]^2 \left[\frac{MVA_{b,new}}{MVA_{b,old}}\right].$

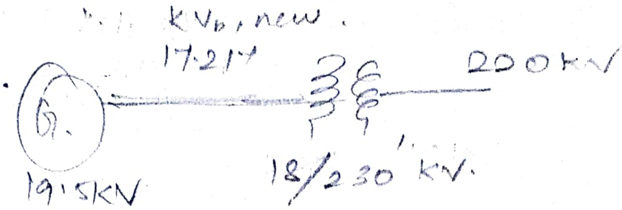
$= 0.15 \times \left[\frac{19.5}{18}\right]^2 \times \left[\frac{150}{120}\right]$

$= 0.22$ p.u.

(C) Given:

$MVA_{b,new} = 100\,MVA.$

$KV_{b,new} = 220\,KV$



Find the base Voltage on L.T side of transformer,

$$KV_b \text{ on LT side of transformer} = KV_b \text{ on HT side} \times \frac{\text{LT Voltage rating}}{\text{HT Voltage rating}}$$

$$= 220 \times \frac{18}{230}$$

$$KV_{b,new} = 17.217\,KV.$$

$$(X_{p.u})\ \text{p.u. Generator reactance} = X_{p.u,old} \times \left[\frac{KV_{b,old}}{KV_{b,new}}\right]^2 \times \left[\frac{MVA_{b,new}}{MVA_{b,old}}\right]$$

$$= 0.15 \left[\frac{19.5}{17.217}\right]^2 \times \left[\frac{100}{120}\right]$$

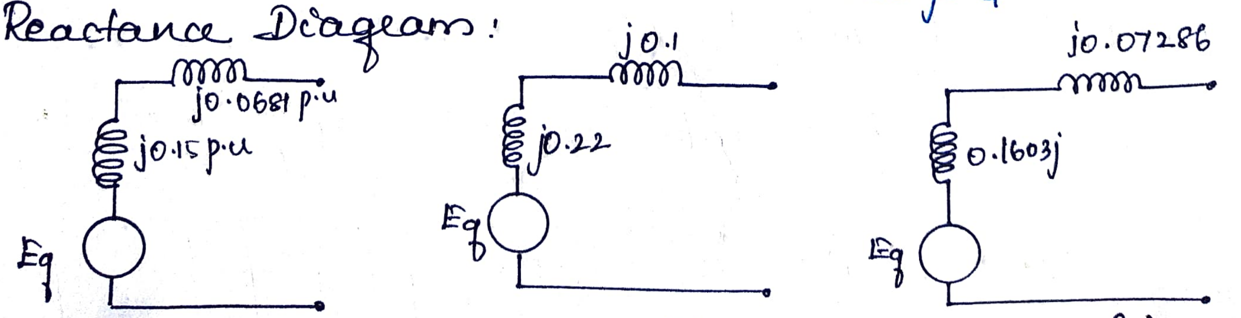
$$= 0.1603\ p.u.$$

$$\text{p.u. Transformer reactance} = X_{p.u,old} \times \left[\frac{KV_{b,old}}{KV_{b,new}}\right]^2 \times \left[\frac{MVA_{b,new}}{MVA_{b,old}}\right]$$

$$= 0.1 \times \left[\frac{18}{17.217}\right]^2 \times \left[\frac{100}{150}\right]$$

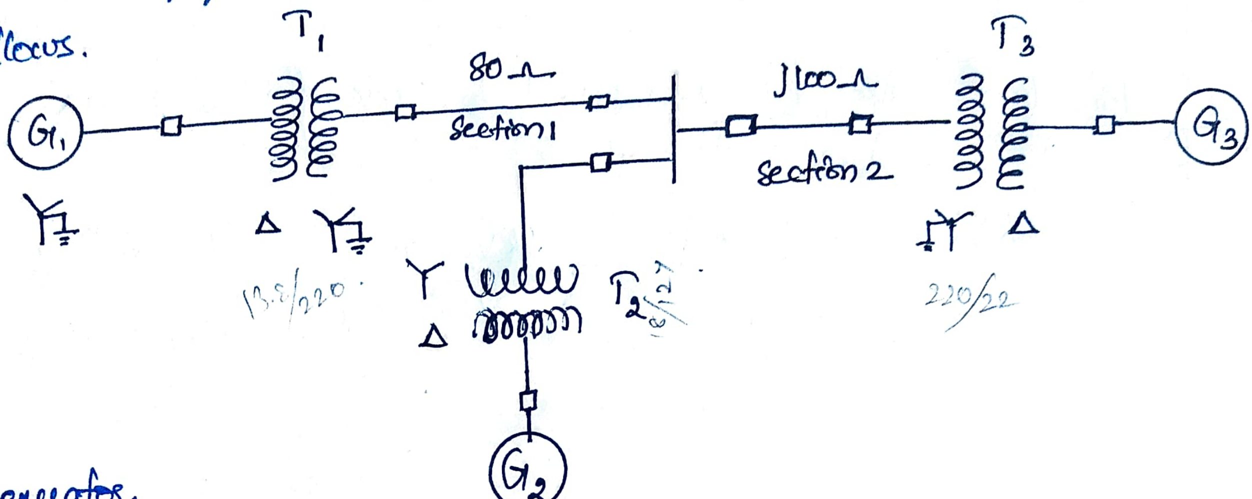
$$= 0.07286\ p.u$$

Reactance Diagram:



(a) (b) (c).

3. The single line diagram of an unloaded power system is shown in figure. The generator and transformers are rated as follows.



Generator,
$G_1 = 20MVA,\ 13.8\ KV,\ X'' = 20\%$.

Generator, $G_2 = 30MVA,\ 18KV,\ X'' = 20\%$.

Generator, $G_3 = 30MVA,\ 20KV,\ X'' = 20\%$.

Transformer, $T_1 = 25MVA,\ 220/13.8\ KV,\ X = 10\%$

Transformer, T_2 = 3 single phase units each rated at 10MVA, 127/18 KV, $X = 10\%$.

Transformer, $T_3 = 35MVA,\ 220/22KV,\ X = 10\%$

Draw the reactance diagram using base of 50 MVA and 13.8kV on the Generator G_1.

<u>Solution:</u>

$$MVA_{b,new} = 50MVA.$$

$$KV_{b,new} = 13.8KV.$$

<u>Reactance of Generator G_1:</u>

$$X_{p.u} \text{ of Generator } G_1 = X_{p.u,old} \times \left[\frac{KV_{b,old}}{KV_{b,new}}\right]^2 \times \left[\frac{MVA_{b,new}}{MVA_{b,old}}\right]$$

$$= 0.2 \times \left[\frac{13.8}{13.8}\right]^2 \times \left[\frac{50}{20}\right]$$

$$= 0.5\ p.u.$$

Reactance of Transformer T_1:

$$X_{p.u} \text{ of Transformer} = X_{p.u,old} \times \left[\frac{KV_{b,old}}{KV_{b,new}}\right]^2 \times \left[\frac{MVA_{b,new}}{MVA_{b,old}}\right]$$

$$= 0.1 \times \left[\frac{13.8}{13.8}\right]^2 \times \left[\frac{50}{25}\right]$$

$$= 0.2 \text{ p.u.}$$

Reactance of Transmission Lines: (sect. 1 & 2).

Base kV on HT side of Transformer T_1 $= \text{Base kV on LT side} \times \dfrac{\text{HT Voltage rating}}{\text{LT Voltage rating}}$.

$$= 13.8 \times \frac{220}{13.8}$$

$$KV_{b,new} = 220 \text{ KV.}$$

Base impedance, Z_b on HT side $= \dfrac{(KV_{b,new})^2}{MVA_{b,new}} = \dfrac{(220)^2}{50} = 968\,\Omega$.

$X_{p.u}$ of section 1 of transmission line $= \dfrac{\text{Actual reactance}}{\text{Base impedance}}$

$$= \frac{80}{968}$$

$$= 0.0826 \text{ p.u.}$$

$X_{p.u.}$ of section 2 of transmission line $= \dfrac{100}{968}$

$$= 0.1033 \text{ p.u.}$$

Reactance of transformer T_2 : KV_b is for G_1 only.

Base KV on LT side of transformer, T_2 $=$ Base KV on HT side $\times \frac{\text{LT Voltage ra}}{\text{HT Voltage ra}}$

$$= 220 \times \frac{18}{220}$$

$$KV_{b,new} = 18 kV.$$

$$X_{p.u} \text{ of transformer, } T_2 = X_{p.u,old} \times \left[\frac{KV_{b,old}}{KV_{b,new}}\right]^2 \times \left[\frac{MVA_{b,new}}{MVA_{b,old}}\right]$$

$$= 0.1 \times \left[\frac{18}{18}\right]^2 \times \left[\frac{50}{3\times10}\right]$$

$$= 0.1666 \text{ p.u}$$

Reactance of Generator, G_2 :

$$X_{p.u} \text{ of Generator} = X_{pu,old} \times \left[\frac{KV_{b,old}}{KV_{b,new}}\right]^2 \times \left[\frac{MVA_{b,new}}{MVA_{b,old}}\right]$$

$$= 0.2 \times \left[\frac{18}{18}\right]^2 \times \left[\frac{50}{30}\right]$$

$$= 0.3333 \text{ p.u}$$

Reactance of Transformer, T_3 :

KV_b on LT side of transformer, T_3 $=$ Base kV on HT side $\times \frac{\text{LT Voltage rating}}{\text{HT Voltage rating}}$

$$= 220 \times \frac{22}{220}$$

$$KV_{b,new} = 22 kV.$$

$X_{p.u}$ of transformer, T_3 $= X_{pu,old} \times \left[\frac{KV_{b,old}}{KV_{b,new}}\right]^2 \times \left[\frac{MVA_{b,new}}{MVA_{b,old}}\right]$

$$= 0.1 \times \left[\frac{22}{22}\right]^2 \times \left[\frac{50}{35}\right]$$

$$= 0.1428 \text{ p.u}$$

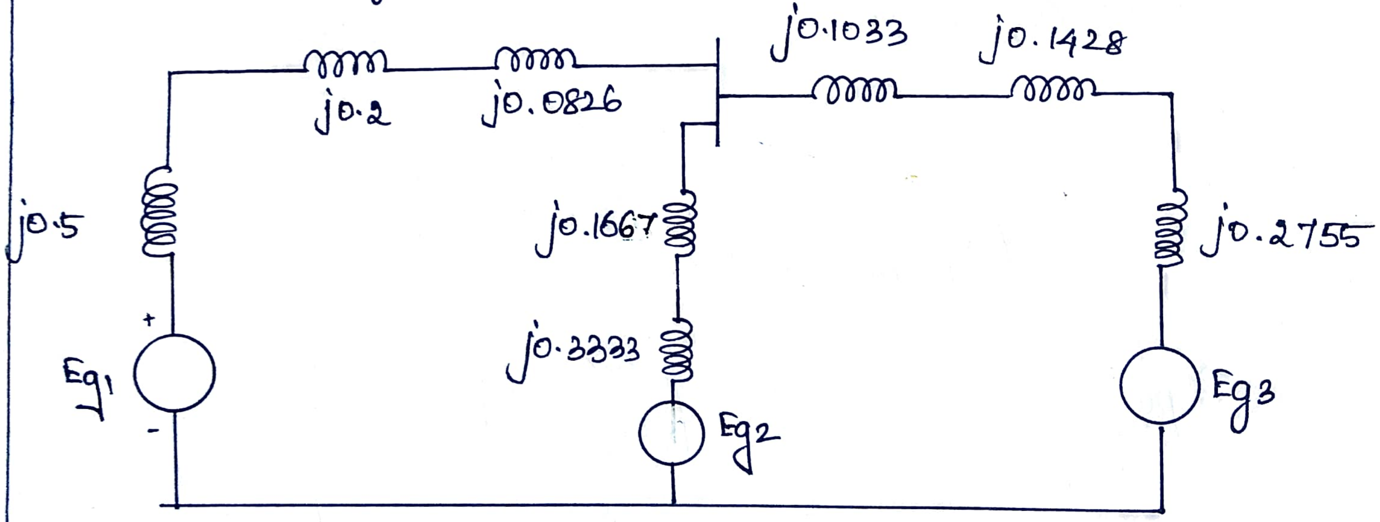
Reactance of Generator, G_3

$$X_{pu,new} = X_{pu,old} \times \left[\frac{KV_{b,old}}{KV_{b,new}}\right]^2 \times \left[\frac{MVA_{b,new}}{MVA_{b,old}}\right]$$

$$= 0.2 \times \left[\frac{20}{22}\right]^2 \times \left[\frac{50}{30}\right]$$

$$= 0.2755 \text{ p.u}$$

Reactance Diagram:



4. Draw the reactance diagram for the power system shown in figure. The ratings of generator, motor and transformers are gr. below. Neglect resistance and use a base of 50 MVA, 138 KV in the 40 Ω line.

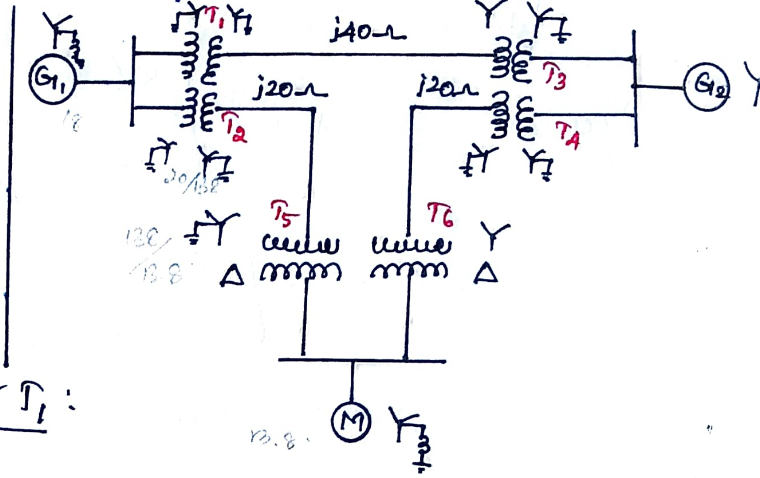
Generator G_1 : 20 MVA, 18 kV, $X'' = 20\%$

Generator G_2 : 20 MVA, 18 kV, $X'' = 20\%$

Syn. Motor : 30 MVA, 13.8 kV, $X'' = 20\%$

3-ϕ Y-Y transformer : 20 MVA, 138/20 kV, $X = 10\%$

3-ϕ Y-Δ Transformer : 15 MVA, 138/13.8 kV, $X = 10\%$



<u>Given :</u>

$MVA_{b,new} = 50\ MVA$

$KV_{b,new} = 138\ kV$

<u>Reactance of Transformer T_1 :</u>

$$\text{Base kV on LT side of txfr} = \text{Base kV on HT side} \times \frac{\text{LT voltage Rating}}{\text{HT voltage rating}}$$

$$= 138 \times \frac{20}{138}$$

$$\boxed{KV_{b,new} = 20\,kV}$$

$$X_{pu,new} = X_{p.u,old} \times \left[\frac{KV_{b,old}}{KV_{b,new}}\right]^2 \times \left[\frac{MVA_{b,new}}{MVA_{b,old}}\right]$$

$$= 0.1 \times \left[\frac{20}{20}\right]^2 \times \left[\frac{50}{20}\right]$$

$$= 0.25\ p.u.$$

<u>Reactance of Generator G_1 :</u>

$$X_{pu,new} = X_{p.u,old} \times \left[\frac{KV_{b,old}}{KV_{b,new}}\right]^2 \times \left[\frac{MVA_{b,new}}{MVA_{b,old}}\right]$$

$$= 0.2 \times \left[\frac{18}{20}\right]^2 \times \left[\frac{50}{20}\right]$$

$$= 0.405 \text{ p.u}$$

Reactance of Transformer, T_2:

$$X_{p.u, new} = X_{p.u, old} \times \left[\frac{KV_{b, old}}{KV_{b, new}}\right]^2 \times \left[\frac{MVA_{b, new}}{MVA_{b, old}}\right]$$

$$= 0.1 \times \left[\frac{20}{20}\right]^2 \times \left[\frac{50}{20}\right]$$

$$= 0.25 \text{ p.u.}$$

Reactance of $j40\Omega$ transmission line:

$$\text{Base impedance} = \frac{(KV_b)^2}{MVA_{b, new}}$$

$$= \frac{138^2}{50}$$

$$= 380.88\Omega.$$

$$X_{pu} = \frac{\text{Actual reactance in } \Omega}{\text{Base impedance in } \Omega.}$$

$$= \frac{40}{380.88}$$

$$= 0.105 \text{ p.u.}$$

Reactance of $j20\Omega$ transmission line:

$$\text{Base kV on HT side of transformer} = \text{Base kV on LT side} \times \frac{\text{HT Voltage rating}}{\text{LT Voltage rating}}$$

$$= 20 \times \frac{138}{20}$$

$KV_{b,new} = 138 \text{ kV}.$

$$\text{Base impedance} = \frac{(KV_{b,new})^2}{MVA_{b,new}}$$

$$= \frac{138^2}{50}$$

$$= 380.88\,\Omega.$$

$$X_{p.u} = \frac{\text{Actual reactance } (\Omega)}{\text{Base impedance } (\Omega)}$$

$$= \frac{20}{380.88}$$

$$= 0.0525 \text{ p.u}$$

Reactance of Transformer, T_5:

$$X_{p.u,new} = X_{p.u,old} \times \left[\frac{KV_{b,old}}{KV_{b,new}}\right]^2 \times \left[\frac{MVA_{b,new}}{MVA_{b,old}}\right]$$

$$= 0.1 \times \left[\frac{138}{138}\right]^2 \times \left[\frac{50}{15}\right]$$

$$= 0.3333 \text{ p.u.}$$

Reactance of synchronous motor:

$$\text{Base KV on LT side of transformer} = KV_b \text{ on HT side} \times \frac{\text{LT voltage rating}}{\text{HT Voltage rating}}$$

$$= 138 \times \frac{13.8}{138}$$

$$KV_{b,new} = 13.8 \text{ KV}.$$

$$X_{p.u,new} \text{ of motor} = X_{p.u,old} \times \left[\frac{KV_{b,old}}{KV_{b,new}}\right]^2 \times \left[\frac{MVA_{b,new}}{MVA_{b,old}}\right]$$

$$= 0.2 \times \left[\frac{13.8}{13.8}\right]^2 \times \left[\frac{50}{30}\right]$$

$$= 0.3333 \text{ p.u.}$$

Reactances of T_6, T_4, T_3 and G_2 :

1. The transformer T_6 is identical to T_5.

$\therefore X_{p.u}$ of T_6 = 0.3333 p.u.

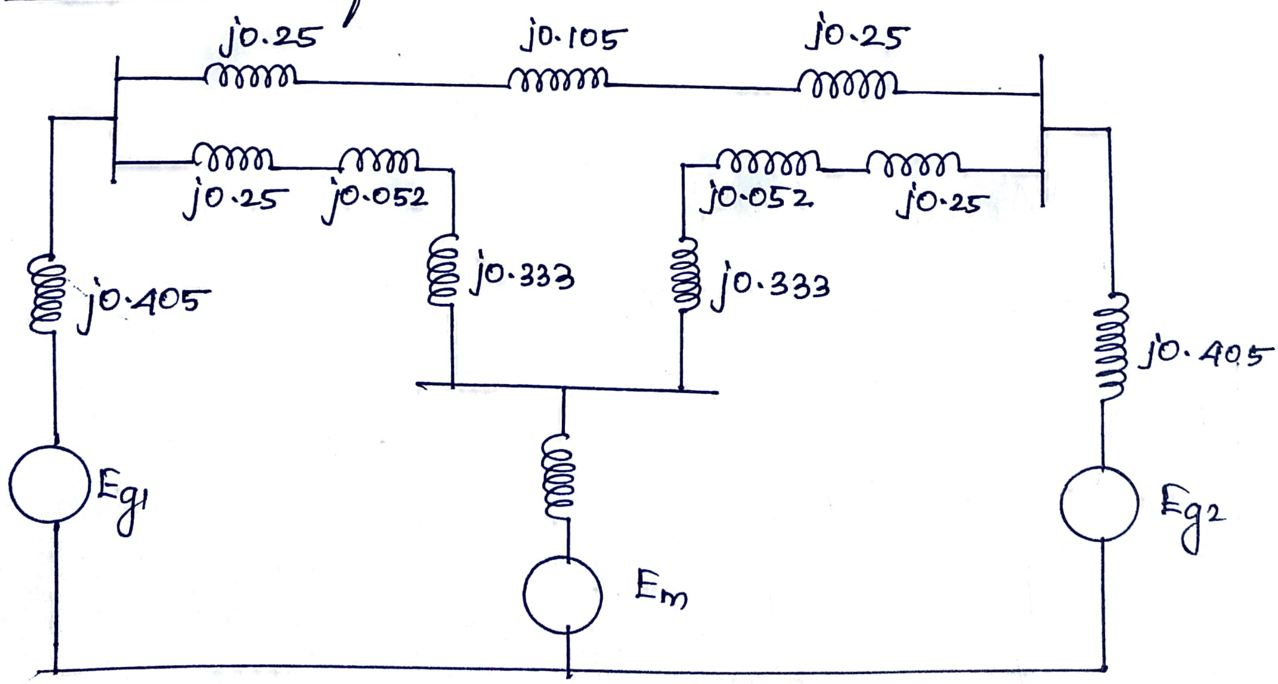
Transformers T_1, T_2, T_3 and T_4 are identical.

$\therefore X_{p.u}$ of T_1, T_2, T_3 & T_4 = 0.25 p.u.

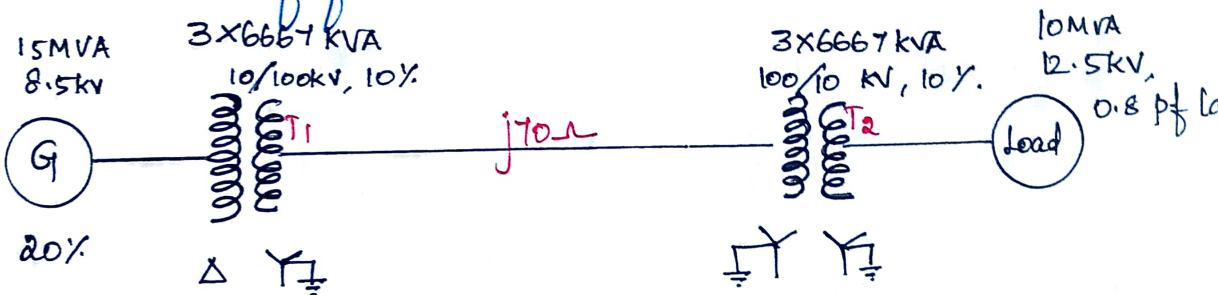
Generator G_2 is identical to G_1.

$X_{p.u}$ of G_2 = 0.405 p.u.

Reactance Diagram :



5. A 15MVA, 8.5KV, 3-ϕ generator has a subtransient reactance of 20%. It is connected through a Δ-Y transformer to a high voltage transmission line having a total series reactance of 70Ω. The load end of the line has Y-Y step down transformer. Both transformer banks are composed of single phase transformers connected for a 3-ϕ operation. Each of 3 transformers composing 3-ϕ Bank is rated 6667 kVA 10/100kV, with reactance of 10%. The load represented as impedance is drawing 10MVA at 12.5kV and 0.8 pf lagging. Draw single line diagram. Choose a base of 10MVA, 12.5kV in the load ckt & draw reactance diagram. Det. voltage at terminals of generator.



Solution:

$$MVA_{b,new} = 10MVA.$$

$$KV_{b,new} = 12.5kV.$$

Load:

$$Pf = 0.8 \text{ lag}$$

$$\cos\phi = 0.8 \text{ lag}.$$

$$\phi = -36.86^\circ.$$

Complex load power = $10\angle -36.86°$ MVA.

p.u. Value of load (power) $= \dfrac{\text{Actual MVA}}{\text{Base MVA}}$

$= \dfrac{10\angle -36.86°}{10}$

$= 1\angle -36.86°.$

p.u. value of load voltage $= \dfrac{\text{Actual Voltage}}{\text{Base Voltage}}$

$= \dfrac{12.5 kV}{12.5 kV}$

$= 1$ p.u.

$I = \dfrac{P\ (p.u)}{V\ (p.u)}.$

$= \dfrac{1\angle -36.86°}{1}.$

$I (p.u) = 1\angle -36.86°.$

Reactance of transformer: (T_2)

$$X_{p.u, new} = X_{p.u, old} \times \left[\frac{KV_{b, old}}{KV_{b, new}}\right]^2 \times \left[\frac{MVA_{b, new}}{MVA_{b, old}}\right]$$

$$= 0.1 \times \left[\frac{10\times\sqrt{3}}{12.5}\right]^2 \times \left[\frac{10}{20}\right]$$

$= 0.0959$ p.u.

$MVA_{b, old}$
$= 3 \times 6667$ KVA
$= 20{,}000$ KVA.
$= 20$ MVA.

Reactance of Transmission line:

Base kV on HT side of transformer $=$ Base kV on LT side $\times \dfrac{\text{HT Voltage rating}}{\text{LT Voltage rating}}$

$$= 12.5 \times \frac{100\times\sqrt{3}}{17.32}$$

$$KV_{b,new} = 125 KV.$$

Base impedance, $$Z_b = \frac{(KV_{b,new})^2}{MVA_{,b,new}}$$

$$= \frac{(125)^2}{10}$$

$$= 1562.5 \Omega.$$

$$\text{p.u reactance} = \frac{\text{Actual impedance } (\Omega)}{\text{Base impedance } (\Omega)}$$

$$= \frac{70}{1562.5}$$

$$= 0.0448 \text{ p.u}$$

<u>Reactance of Transformer, T_1</u> :

$$X_{p.u,new} = X_{pu,old} \times \left[\frac{KV_{b,old}}{KV_{b,new}}\right]^2 \times \left[\frac{MVA_{b,new}}{MVA_{b,old}}\right]$$

$$= 0.1 \times \left[\frac{173.2}{125}\right]^2 \times \left[\frac{10}{20}\right]$$ ← 3×1667

$$= 0.0959 \text{ p.u.}$$

<u>Reactance of Generator, G_1</u> :

Base KV on LT side $= KV_b$ on HT side $\times \frac{\text{LT Voltage rating}}{\text{HT Voltage rating}}$.

$$= 173.2 \times \frac{10}{\sqrt{3} \times 100}$$

$$= 7.217 \text{ KV.}$$

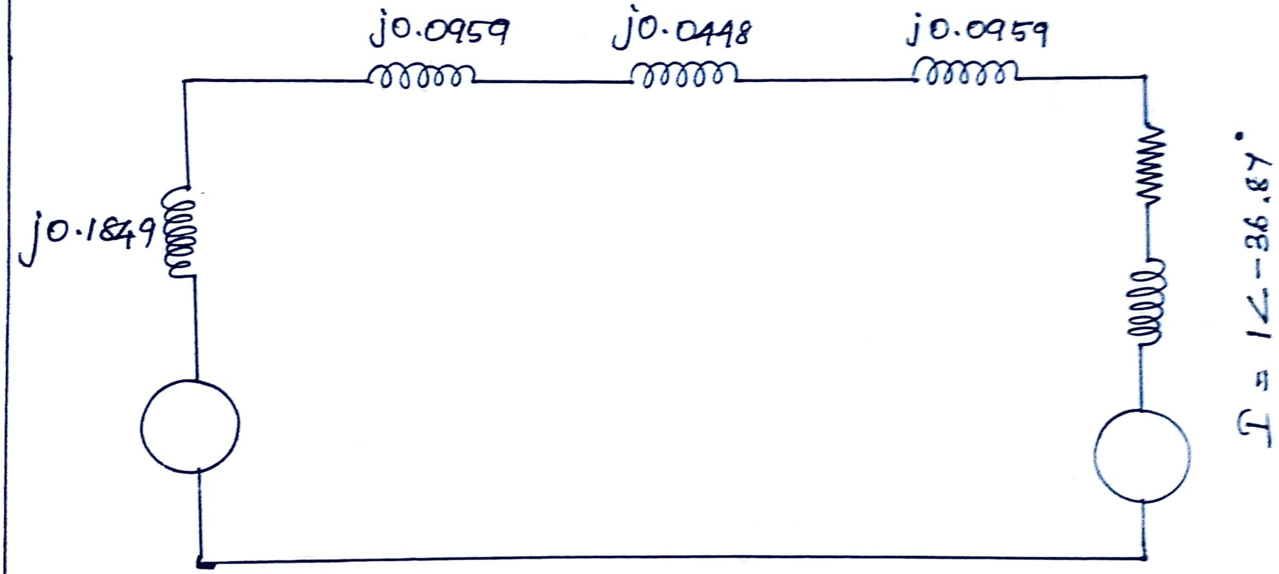
$$KV_{b,new} = 7.217 \text{ KV.}$$

$$X_{p.u,new} = X_{p.u,old} \times \left[\frac{KV_{b,old}}{KV_{b,new}}\right]^2 \times \left[\frac{MVA_{b,new}}{MVA_{b,old}}\right]$$

$$= 0.2 \times \left[\frac{8.5}{7.217}\right]^2 \times \left[\frac{10}{15}\right]$$

$$= 0.1849 \text{ p.u.}$$

Reactance Diagram:



Voltage at the terminals of generator:

From above fig,

$$V_t = V + I(j0.0959 + j0.0448 + j0.0959)$$

$$= 1.0 + (1\angle -36.87^\circ)(j0.0959 + j0.0448 + j0.0959)$$

$$= 1.0 + (1\angle -36.87^\circ)(0.2366j)$$

$= 1 + 1\angle -36.87^\circ \times 0.2366 \angle 90^\circ.$

$= 1 + 0.2366 \angle 53.13^\circ.$

$= 1 + 0.1421 + j0.1894$

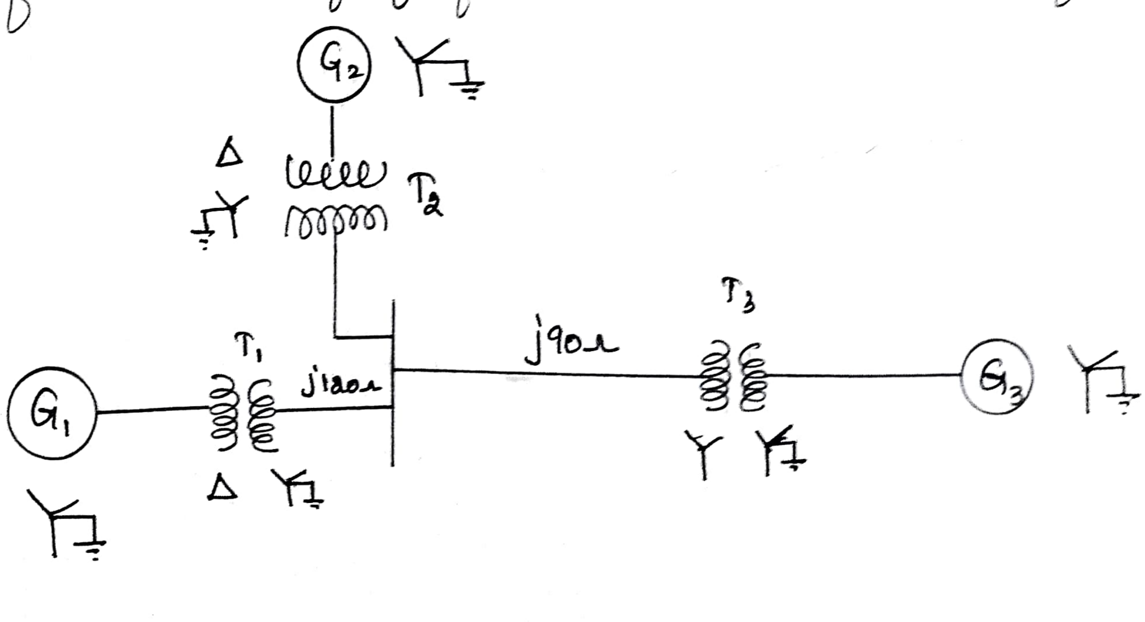
$= 1.1421 + j0.1894 = 1.1577 \angle 9.4^\circ$ p.u

Average value of generator terminal Voltage } = p.u Value of Voltage } × Base kV on LT side of transformer T_1

$= 1.1577 \angle 9.4^\circ \times 7.217$

$= 8.355 \angle 9.4^\circ$ kV.

8. Figure shows the single line diagram of unloaded three generator power system with interconnection between the generators by means of three transformers and a transmission line with two sections with their impedances marked on the diagram. The ratings of generators & transformers are as follows



Generator	MVA	kV	Reactance in p.u.
1	25	6.6	0.2
2	15	6.6	0.15
3	30	13.2	0.15

Transformer 1 : 30MVA, 6.9Δ – 115Y kV, X = 10%

Transformer 2 : 15MVA, 6.9Δ – 115Y kV, X = 10%

Transformer 3 : Single phase units, each rated 10MVA, 6.9/69 kV, X = 10%.

Draw an impedance diagram and mark all values in p.u choosing a base of 30MVA, 6.6 kV in generator 1 circuit.

Bus admittance Matrix :

(The meeting point of various components in a power system is called bus) The buses can be treated as nodes & the voltages of all buses can be solved by conventional node analysis technique.

Let $V_1, V_2, V_3 \ldots V_n$: Node voltages of nodes $1, 2, 3 \ldots n$.

$I_{11}, I_{22}, I_{33} \ldots I_{nn}$: Sum of current sources connected to nodes $1, 2, 3 \ldots n$.

N : number of major or principal nodes.

Since the voltage of the node can be measured only with respect to a reference point, one of the node is considered as reference node. So the n/w will have $(N-1)$ independant voltages.

* The solution can be obtained by KCL or KVL equations.
* For N-bus system, the KCL can be written as,

$$Y_{11}V_1 + Y_{12}V_2 + Y_{13}V_3 + \ldots Y_{1n}V_n = I_{11}$$

$$Y_{21}V_1 + Y_{22}V_2 + Y_{23}V_3 + \ldots Y_{2n}V_n = I_{22}$$

$$Y_{31}V_1 + Y_{32}V_2 + Y_{33}V_3 + \ldots Y_{3n}V_n = I_{33}$$

$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

$$Y_{n1}V_1 + Y_{n2}V_2 + Y_{n3}V_3 + \ldots Y_{nn}V_n = I_{nn}$$

The above equations can be written in matrix form as,

$$\begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & \cdots & Y_{1n} \\ Y_{21} & Y_{22} & Y_{23} & \cdots & Y_{2n} \\ \vdots & \vdots & \vdots & & \vdots \\ Y_{n1} & Y_{n2} & Y_{n3} & \cdots & Y_{nn} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix} = \begin{bmatrix} I_{11} \\ I_{22} \\ \vdots \\ I_{nn} \end{bmatrix}$$

Eqn. ② can be written as,

$$YV = I \qquad \text{--- (3)}$$

In power system, Y-matrix is designated as 'Y_{bus}' and called bus matrix admittance matrix.

i.e, $$Y_{bus}V = I \qquad \text{--- (4)}$$

where, Y_{bus} = Bus admittance matrix of order (n×n).

V = Bus Voltage matrix of order (n×1)

I = Current sources matrix of order (n×1).

n : no. of independant buses in the s/m.

Bus admittance matrix, Y_{bus} is given by,

$$Y_{bus} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & \cdots & Y_{1n} \\ Y_{21} & Y_{22} & Y_{23} & \cdots & Y_{2n} \\ Y_{31} & Y_{32} & Y_{33} & \cdots & Y_{3n} \\ \vdots & \vdots & \vdots & & \\ Y_{n1} & Y_{n2} & Y_{n3} & \cdots & Y_{nn} \end{bmatrix} \qquad \text{--- 5}$$

* Y_{bus} is symmetrical about diagonal.
* Y_{11}, Y_{22}, Y_{33} ... are called Self admittances.
* $Y_{12}, Y_{13}, Y_{21}, Y_{23}$ ... are called Mutual admittances. (other than self admittances).
* Y_{jj} = Sum of all admittances connected to bus-j.
* Y_{jk} = Negative sum of all admittances connected b/w bus-j and bus-k

Also, $Y_{jk} = Y_{kj}$

Solution of Bus Voltages:

WKT,

$$Y_{bus} V = I$$

$$V = Y_{bus}^{-1} I \quad \text{———— (6)}$$

$$Y_{bus}^{-1} = \frac{\text{Adjoint of } Y_{bus}}{\text{Determinant of } Y_{bus}}$$

$$\text{Adjoint of } Y_{bus} = \begin{bmatrix} \Delta_{11} & \Delta_{12} & \Delta_{13} & \cdots & \Delta_{1n} \\ \Delta_{21} & \Delta_{22} & \Delta_{23} & \cdots & \Delta_{2n} \\ \Delta_{31} & \Delta_{32} & \Delta_{33} & \cdots & \Delta_{3n} \\ \vdots \\ \Delta_{n1} & \Delta_{n2} & \Delta_{n3} & \cdots & \Delta_{nn} \end{bmatrix}^T$$

$$= \begin{bmatrix} \Delta_{11} & \Delta_{21} & \Delta_{31} & \cdots & \Delta_{n1} \\ \Delta_{12} & \Delta_{22} & \Delta_{32} & \cdots & \Delta_{n2} \\ \Delta_{13} & \Delta_{23} & \Delta_{33} & \cdots & \Delta_{n3} \\ \vdots & \vdots & \vdots & & \vdots \\ \Delta_{1n} & \Delta_{2n} & \Delta_{3n} & \cdots & \Delta_{nn} \end{bmatrix} \quad \text{———— (7)}$$

where, Δ_{ij} = Cofactor of Y_{jk}

$$V = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ \vdots \\ V_n \end{bmatrix} \qquad I = \begin{bmatrix} I_{11} \\ I_{22} \\ I_{33} \\ \vdots \\ I_{nn} \end{bmatrix} \quad \text{———— (8)}.$$

Subs. eq^{n}s (7) & (8) in (6).

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ \vdots \\ V_n \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} \Delta_{11} & \Delta_{21} & \Delta_{31} & \cdots & \Delta_{n1} \\ \Delta_{12} & \Delta_{22} & \Delta_{32} & \cdots & \Delta_{n2} \\ \Delta_{13} & \Delta_{23} & \Delta_{33} & \cdots & \Delta_{n3} \\ \vdots & \vdots & \vdots & & \vdots \\ \Delta_{1n} & \Delta_{2n} & \Delta_{3n} & \cdots & \Delta_{nn} \end{bmatrix} \begin{bmatrix} I_{11} \\ I_{22} \\ I_{33} \\ \vdots \\ I_{nn} \end{bmatrix}$$

which gives.

$$V_1 = \frac{1}{\Delta}\left[\Delta_{11} I_{11} + \Delta_{21} I_{22} + \Delta_{31} I_{33} + \cdots \Delta_{n1} I_{nn}\right]$$

$$V_2 = \frac{1}{\Delta}\left[\Delta_{12} I_{11} + \Delta_{22} I_{22} + \Delta_{32} I_{33} + \cdots \Delta_{n2} I_{nn}\right]$$

$$V_3 = \frac{1}{\Delta}\left[\Delta_{13} I_{11} + \Delta_{23} I_{22} + \Delta_{33} I_{33} + \cdots \Delta_{n3} I_{nn}\right]$$

$$\vdots \qquad\qquad \vdots$$

$$V_n = \frac{1}{\Delta}\left[\Delta_{1n} I_{11} + \Delta_{2n} I_{22} + \Delta_{3n} I_{33} + \cdots \Delta_{nn} I_{nn}\right].$$

In general, the k^{th} bus voltage is g.v. by,

$$V_K = \frac{1}{\Delta}\left[\Delta_{1K} I_{11} + \Delta_{2K} I_{22} + \Delta_{3K} I_{33} + \cdots \Delta_{nK} I_{nn}\right]$$

$$V_K = \frac{1}{\Delta} \sum_{j=1}^{n} \Delta_{jK} I_{jj} \quad \text{———————} \quad (9)$$

Δ : Determinant of Y_{bus} matrix.

I_{jj} : Sum of current sources injecting current to node

Δ_{jk} : Cofactor of element Y_{jk} of bus admittance m

(or).

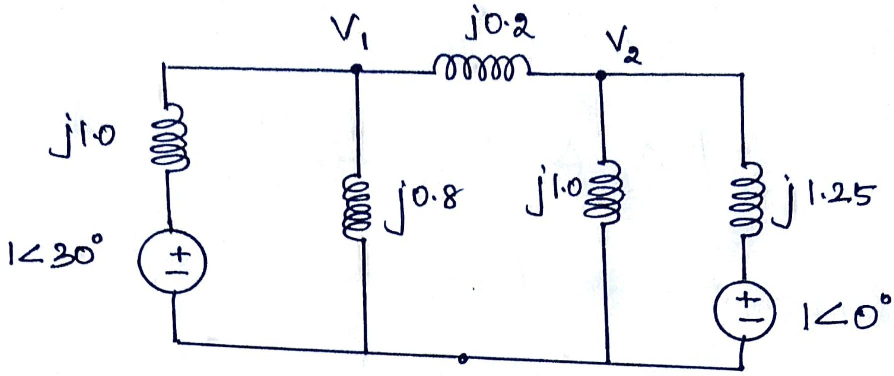
$$V_K = \frac{\Delta_K}{\Delta}.$$

Δ_K : det. of Y_{bus} matrix after replacing k^{th} column by current source vector I.

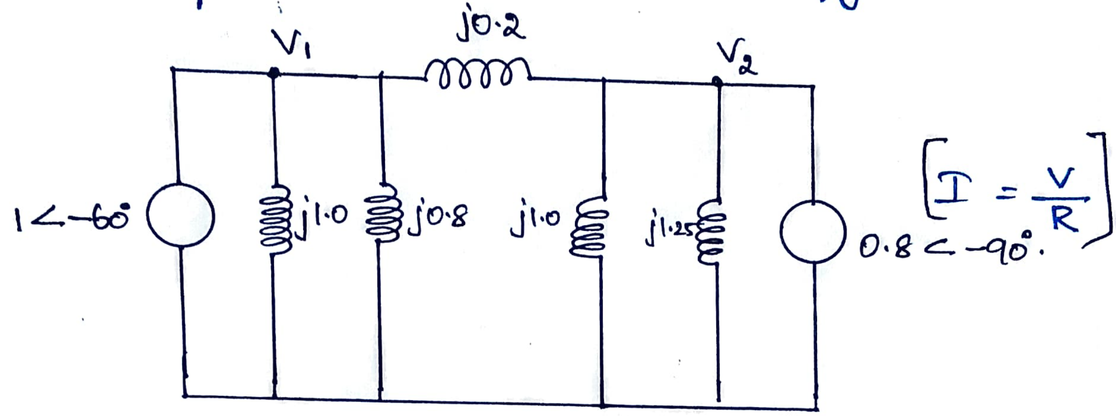
Problems

1. Solve the node voltages V_1 and V_2 in the network shown. The voltages & impedances are in p.u.

step 1: Convert voltage source to current source.



After converting Vge source to current source fig. becomes,



The equation is formed as,

$$\begin{bmatrix} \frac{1}{j1.0}+\frac{1}{j0.8}+\frac{1}{j0.2} & -\frac{1}{j0.2} \\ -\frac{1}{j0.2} & \frac{1}{j0.2}+\frac{1}{j1.0}+\frac{1}{j1.25} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 1\angle -60^\circ \\ 0.8\angle -90^\circ \end{bmatrix}$$

$$\begin{bmatrix} -7.25j & 5j \\ 5j & -6.8j \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 1\angle -60^\circ \\ 0.8\angle -90^\circ \end{bmatrix}$$

To find V_1 & V_2,

$$V_K = \frac{1}{\Delta}\sum_{j=1}^{n} \Delta_{jk} I_{jj}$$

$$\Delta = \begin{vmatrix} -7.25j & 5j \\ 5j & -6.8j \end{vmatrix}$$

$$= -24.3$$

$$V_1 = \frac{1}{\Delta} \sum_{j=1}^{2} \Delta_{j1} \Delta_{jj}^{I}$$

$$V_1 = \frac{1}{\Delta} \left[\Delta_{11} \overset{I_{11}}{\Delta} + \Delta_{21} I_{22} \right].$$

Δ_{11} = cofactor of Y_{11} = $-j6.8$.

Δ_{21} = cofactor of Y_{21} =

Bus (or) Node Elimination:

The buses or nodes which doesnot have any current sources can be eliminated. ~~by~~ Consider a (5X5) Y_{bus} matrix.

$$Y_{bus} V = I$$

ie,

$$\left[\begin{array}{ccc|cc} Y_{11} & Y_{12} & Y_{13} & Y_{14} & Y_{15} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} & Y_{25} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} & Y_{35} \\ \hline Y_{41} & Y_{42} & Y_{43} & Y_{44} & Y_{45} \\ Y_{51} & Y_{52} & Y_{53} & Y_{54} & Y_{55} \end{array}\right] \left[\begin{array}{c} V_1 \\ V_2 \\ V_3 \\ \hline V_4 \\ V_5 \end{array}\right] = \left[\begin{array}{c} I_{11} \\ I_{22} \\ I_{33} \\ \hline I_{44} \\ I_{55} \end{array}\right] \quad \text{---(1)}$$

Let 4 and 5 buses to be eliminated,

let, $K = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix}$ Self + Mutual admittances for the buses to be retained.

$L = \begin{bmatrix} Y_{14} & Y_{15} \\ Y_{24} & Y_{25} \\ Y_{34} & Y_{35} \end{bmatrix}$ only mutual admittances, for the buses to be eliminated

$L^T = \begin{bmatrix} Y_{41} & Y_{42} & Y_{43} \\ Y_{51} & Y_{52} & Y_{53} \end{bmatrix}$ Transpose of L.

$M = \begin{bmatrix} Y_{44} & Y_{45} \\ Y_{54} & Y_{55} \end{bmatrix}$ Self + Mutual admittances for the buses to be eliminated.

$V_A = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$

$V_X = \begin{bmatrix} V_4 \\ V_5 \end{bmatrix}$ Submatrix composed of the voltages of the buses to be eliminated.

$$I_A = \begin{bmatrix} I_{11} \\ I_{22} \\ I_{33} \end{bmatrix}$$

$$I_X = \begin{bmatrix} I_{44} \\ I_{55} \end{bmatrix}$$; Submatrix composed of currents entering the buses to be eliminated.

Using the submatrices given above, matrix equation (1) can be written as,

$$\begin{bmatrix} K & L \\ L^T & M \end{bmatrix} \begin{bmatrix} V_A \\ V_X \end{bmatrix} = \begin{bmatrix} I_A \\ I_X \end{bmatrix} \quad \text{———— (2)}.$$

Eq^n.s are,

$$KV_A + LV_X = I_A \quad \text{———— (3)}$$

$$L^T V_A + MV_X = I_X \quad \text{———— (4)}$$

From, (4).

$$MV_X = I_X - L^T V_A.$$

$$I_X = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$; no current source,

$$MV_X = 0 - L^T V_A$$

$$V_X = -L^T V_A M^{-1} \quad \text{———— (5)}.$$

Subs. Eq^n (5) in (3).

$$KV_A + L\left[-L^T V_A M^{-1}\right] = I_A.$$

$$V_A\left[K - LM^{-1}L^T\right] = I_A$$

$$V_A Y_{bus,new} = I_A.$$

where, $\underline{Y_{bus,new} = K - LM^{-1}L^T}$. ——————— (6)

* Bus elimination can be simplified if one bus is eliminated at a time.

* Consider a bus admittance matrix of order $(n \times n)$ in which n^{th} bus has to be eliminated.

$$Y_{bus} = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1(n-1)} & Y_{1n} \\ Y_{21} & Y_{22} & \cdots & Y_{2(n-1)} & Y_{2n} \\ \vdots & & & & \vdots \\ Y_{(n-1)1} & Y_{(n-1)2} & \cdots & Y_{(n-1)(n-1)} & Y_{(n-1)n} \\ Y_{n1} & Y_{n2} & \cdots & Y_{n(n-1)} & Y_{nn} \end{bmatrix}$$

$$K = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1(n-1)} \\ Y_{21} & Y_{22} & \cdots & Y_{2(n-1)} \\ \vdots & & & \\ Y_{(n-1)1} & Y_{(n-2)2} & \cdots & Y_{(n-1)(n-1)} \end{bmatrix} ; \quad L = \begin{bmatrix} Y_{1n} \\ Y_{2n} \\ \vdots \\ Y_{(n-1)n} \end{bmatrix}$$

$$L^T = \begin{bmatrix} Y_{n1} & Y_{n2} & \cdots & Y_{n(n-1)} \end{bmatrix}$$

$$M = Y_{nn} \quad ; \quad M^{-1} = \frac{1}{Y_{nn}}.$$

From eq:(6) we get,

$$Y_{bus,new} = K - LM^{-1}L^T.$$

Substitute the above defined values in above eqn.,

$$Y_{bus,new} = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1(n-1)} \\ Y_{21} & Y_{22} & \cdots & Y_{2(n-1)} \\ \vdots & & & \\ Y_{(n-1)1} & Y_{(n-2)2} & \cdots & Y_{(n-1)n-1} \end{bmatrix} - \begin{bmatrix} Y_{1n} \\ Y_{2n} \\ \vdots \\ Y_{(n-1)n} \end{bmatrix} \begin{bmatrix} Y_{n1} & Y_{n2} & \cdots & Y_{n(n-1)} \end{bmatrix} \begin{bmatrix} \frac{1}{Y_{nn}} \end{bmatrix}$$

$$Y_{bus,new} = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1(n-1)} \\ Y_{21} & Y_{22} & \cdots & Y_{2(n-1)} \\ \vdots & & & \\ Y_{(n-1)1} & Y_{(n-2)2} & \cdots & Y_{(n-1)(n-1)} \end{bmatrix} - \frac{1}{Y_{nn}} \begin{bmatrix} Y_{1n}Y_{n1} & Y_{1n}Y_{n2} & \cdots & Y_{1n}Y_{n(n-1)} \\ Y_{2n}Y_{n1} & Y_{2n}Y_{n2} & \cdots & Y_{2n}Y_{n(n-1)} \\ \vdots & & & \\ Y_{(n-1)n}Y_{n1} & Y_{(n-1)n}Y_{n2} & \cdots & Y_{(n-1)n}Y_{n(n-1)} \end{bmatrix}$$

The element, Y_{jk} can be written as,

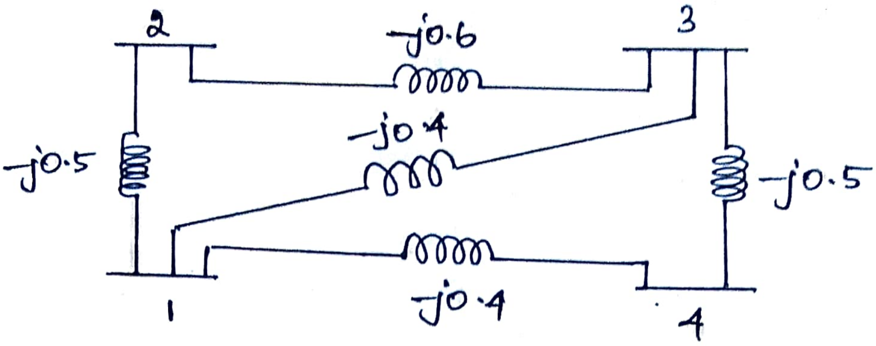
$$\boxed{Y_{jk,new} = Y_{jk} - \frac{Y_{jn}Y_{nk}}{Y_{nn}}}$$

$j = 1,2,3 \cdots (n-1)$

$k = 1,2,3 \cdots (n-1)$

$Y_{jk}, Y_{jn}, Y_{nk}, Y_{nn}$ ← elements of given bus admittance matrix

For the n/w shown in fig. form the bus admittance matrix. Determine the reduced admittance matrix by eliminating node 4. The values are marked in p.u.



Y_{bus} matrix is,

$$Y_{bus} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix}$$

$$= \begin{bmatrix} -(j0.5+0.4j+0.4j) & j0.5 & j0.4 & j0.4 \\ j0.5 & -(j0.5+j0.6) & j0.6 & 0 \\ j0.4 & j0.6 & -(j0.6+0.5j+0.4j) & 0.5j \\ j0.4 & 0 & j0.5 & -(j0.5+j0.4) \end{bmatrix}$$

$$Y_{bus} = \begin{bmatrix} -j1.3 & j0.5 & j0.4 & j0.4 \\ j0.5 & -j1.1 & j0.6 & 0 \\ j0.4 & j0.6 & -j1.5 & j0.5 \\ j0.4 & 0 & j0.5 & -j0.9 \end{bmatrix}$$

The elements of new bus admittance matrix after eliminating 4th row & 4th column is g. by,

$$Y_{jk,new} = Y_{jk} - \frac{Y_{jn} Y_{nk}}{Y_{nn}} \quad ; \quad n = 4, \; j = 1,2,3. \; k = 1,2,3.$$

Bus is symmetrical, so, $Y_{kj,new} = Y_{jk,new}$

$$Y_{11,new} = Y_{11} - \frac{Y_{14} Y_{41}}{Y_{44}}$$

$$= j1.3 - \frac{(j0.4)(j0.4)}{-j0.9}$$

$$= -j1.12.$$

$$Y_{12,new} = Y_{12} - \frac{Y_{14} Y_{42}}{Y_{44}} = j0.5 - \frac{j0.4 \times 0}{-j0.9} = j0.5$$

$$Y_{13,new} = Y_{13} - \frac{Y_{14} Y_{43}}{Y_{44}} = j0.4 - \frac{j0.4 \times j0.5}{-j0.9} = 0.622j.$$

$$Y_{21,new} = Y_{21} - \frac{Y_{24} Y_{41}}{Y_{44}} = j0.5 - \frac{0 \times j0.4}{-j0.9} = j0.5$$

$$Y_{22,new} = Y_{22} - \frac{Y_{24} Y_{42}}{Y_{44}} = -j1.1 - \frac{0 \times 0}{-j0.9} = -j1.1$$

$$Y_{23,new} = Y_{23} - \frac{Y_{24} Y_{43}}{Y_{44}} = j0.6 - \frac{0 \times j0.5}{-j0.9} = j0.6$$

$$Y_{31,new} = Y_{31} - \frac{Y_{34} Y_{41}}{Y_{44}} = Y_{13,new} = 0.622j.$$

$$Y_{32,new} = Y_{23,new} = j0.6$$

$$Y_{33,new} = Y_{33} - \frac{Y_{34} Y_{43}}{Y_{44}} = -j1.5 - \frac{j0.5 \times j0.5}{-j0.9} = -j1.22$$

The reduced bus admittance matrix after eliminating 4th row is,

$$Y_{bus} = \begin{bmatrix} -j1.12 & j0.5 & j0.622 \\ j0.5 & -j1.1 & j0.6 \\ j0.622 & j0.6 & -j1.22 \end{bmatrix}$$

---

2. Eliminate buses 3 and 4 in the given bus admittance matrix and form new bus admittance matrix.

$$Y_{bus} = \begin{bmatrix} -j9.8 & 0.0 & j4.0 & j5.0 \\ 0 & -j8.3 & j2.5 & j5 \\ j4 & j2.5 & -j14 & j8 \\ j5 & j5 & j8 & -j18 \end{bmatrix}$$

First lets eliminate 4th bus, $Y_{nn} = Y_{44} = -j18$

$$Y_{jk,new} = Y_{jk} - \frac{Y_{jn} Y_{nk}}{Y_{nn}}$$

$$Y_{11,new} = Y_{11} - \frac{Y_{14} Y_{41}}{Y_{44}} = -j9.8 - \frac{j5 \times j5}{-j18.0} = -8.4j.$$

$$Y_{12,new} = Y_{12} - \frac{Y_{14} Y_{42}}{Y_{44}} = 0 - \frac{j5 \times j5}{-j18} = j1.388.$$

Same

$$Y_{13,new} = Y_{13} - \frac{Y_{14} Y_{43}}{Y_{44}} = j4 - \frac{j5 \times j8}{-j18} = j6.22$$

$$Y_{21,new} = Y_{21} - \frac{Y_{24} Y_{41}}{Y_{44}} = 0 - \frac{j5 \times j5}{-j18} = j1.388$$

1.22

$$Y_{22,new} = Y_{22} - \frac{Y_{24} Y_{42}}{Y_{44}} = -j8.3 - \frac{j5 \times j5}{-j18} = -6.91j$$

$$Y_{23,new} = Y_{23} - \frac{Y_{24}Y_{43}}{Y_{44}} = j2.5 - \frac{j5 \times j8}{-j18} = j4.72$$

$$Y_{31,new} = Y_{13,new} = j6.22$$

$$Y_{32,new} = Y_{23,new} = j4.72$$

$$Y_{33,new} = Y_{33} - \frac{Y_{34}Y_{43}}{Y_{44}} = -j14 - \frac{j8 \times j8}{-j18} = -10.44j.$$

The reduced bus admittance matrix after eliminating 4th node Y

$$Y_{bus} = \begin{bmatrix} -8.411j & j1.3889 & j6.22 \\ j1.388 & -j6.911 & j4.722 \\ j6.22 & j4.722 & -j10.44 \end{bmatrix}$$

Elimination of node 3 : $Y_{nn} = Y_{33} = -j10.44$.

$$Y_{jk,new} = Y_{jk} - \frac{Y_{jn}Y_{nk}}{Y_{nn}} \; ; \; j = 1,2 \; ; \; k = 1,2 \; , \; n = 3$$

$$Y_{11,new} = Y_{11} - \frac{Y_{13}Y_{31}}{Y_{33}} = -j8.411 - \frac{j6.22 \times j6.222}{-j10.44}$$

$$= -j4.7$$

$$Y_{12,new} = Y_{12} - \frac{Y_{13}Y_{23}}{Y_{33}} = j1.388 - \frac{j6.22 \times j4.72}{-j10.44} = j4.2$$

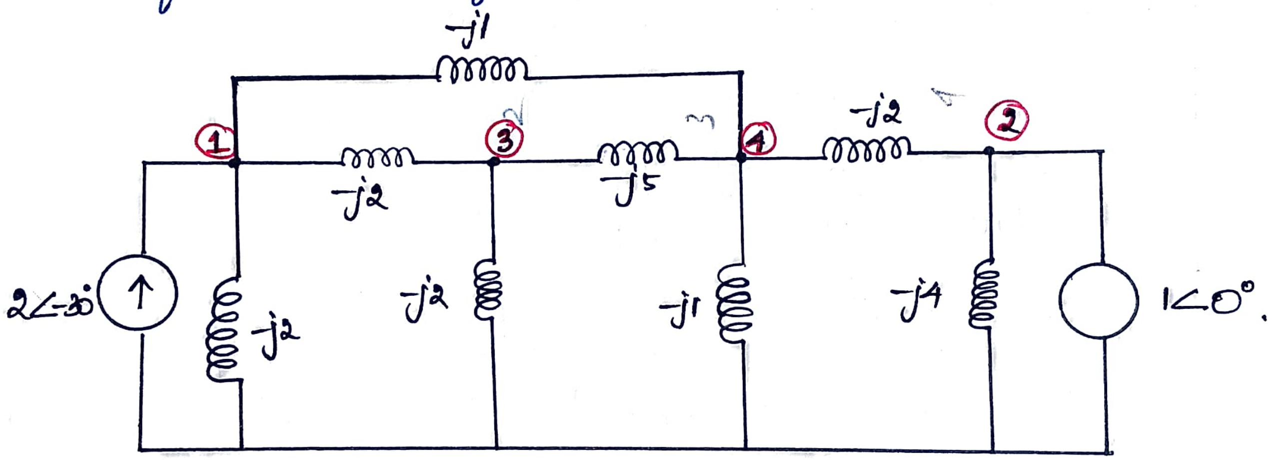
$$Y_{21,new} = Y_{12,new} = j4.2$$

$$Y_{22,new} = Y_{22} - \frac{Y_{23}Y_{32}}{Y_{33}} = -j6.91 - \frac{j4.72 \times j4.722}{-j10.44} = -4.7$$

The reduced bus admittance matrix after eliminating 3 & 4 is,

$$Y_{bus} = \begin{bmatrix} -j4.704 & j4.201 \\ j4.2011 & -j4.776 \end{bmatrix}$$

3. Determine the bus admittance matrix of the system whose reactance diagram is shown in fig. The currents & admittances are given in p.u. Determine the reduced bus admittance matrix after eliminating node 3.



The bus admittance matrix is formed as follows,

$$Y_{bus} = \begin{bmatrix} -j1-j2-j2 & 0 & j2 & j1 \\ 0 & -j2-j4 & 0 & j2 \\ j2 & 0 & -j2-j5-j2 & j5 \\ j1 & j2 & j5 & -j1-j5-j1-j2 \end{bmatrix}$$

$$= \begin{bmatrix} -j5 & 0 & j2 & j1 \\ 0 & -j6 & 0 & j2 \\ j2 & 0 & -j9 & j5 \\ j1 & j2 & j5 & -j9 \end{bmatrix}$$

For eliminating node-3, bus admittance matrix is rearranged by interchanging row-3 & row-4, and then interchanging column-3 & column-4.

① Interchange row-3 & row-4.

$$Y_{bus} = \begin{bmatrix} -j5 & 0 & j2 & j1 \\ 0 & -6j & 0 & j2 \\ j1 & j2 & j5 & -j9 \\ j2 & 0 & -j9 & j5 \end{bmatrix}$$

② Interchange column-3 & column-4

$$Y_{bus} = \begin{bmatrix} -j5 & 0 & j1 & j2 \\ 0 & -6j & j2 & 0 \\ j1 & j2 & -j9 & j5 \\ j2 & 0 & j5 & -j9 \end{bmatrix}$$

③ Now The 4^{th} column & 4^{th} row can be eliminated.

Formula:

$$Y_{jk,new} = Y_{jk} - \frac{Y_{jn} Y_{nk}}{Y_{nn}} \quad ; \text{where } n=4; \; j=1,2,3 \; \& \; k=1,2,3$$

$$Y_{11,new} = Y_{11} - \frac{Y_{14} Y_{41}}{Y_{44}} = -j5 - \frac{(j2)(j2)}{-j9} = -j4.556$$

$$Y_{12,new} = Y_{12} - \frac{Y_{14} Y_{42}}{Y_{44}} = 0 - \frac{j2 \times 0}{-j9} = 0.$$

$$Y_{13,new} = Y_{13} - \frac{Y_{14} Y_{43}}{Y_{44}} = j1 - \frac{(j2)(j5)}{-j9} = j2.111$$

$$Y_{21,new} = Y_{12,new} = Y_{21} - \frac{Y_{24} Y_{41}}{Y_{44}} = 0$$

$$Y_{22,new} = Y_{22} - \frac{Y_{24} Y_{42}}{Y_{44}} = -j6 - \frac{0 \times 0}{j9} = -j6.$$

$$Y_{23,new} = Y_{23} - \frac{Y_{24} Y_{43}}{Y_{44}} = j2 - \frac{0 \times j5}{j9} = j2.$$

$$Y_{31,new} = Y_{13,new} = Y_{31} - \frac{Y_{34} Y_{41}}{Y_{44}} = j2.111$$

$$Y_{32,new} = Y_{23,new} = Y_{32} - \frac{Y_{34} Y_{42}}{Y_{44}} = j2.$$

$$Y_{33,new} = Y_{33} - \frac{Y_{34} Y_{43}}{Y_{44}} = -j9 - \frac{j5 \times j5}{-j9} = -j6.222.$$

The reduced bus admittance matrix after eliminating bus-3 is g.- by,

$$Y_{bus} = \begin{bmatrix} -j4.556 & 0 & j2.111 \\ 0 & -j6 & j2 \\ j2.111 & j2 & -j6.222 \end{bmatrix}$$

Bus impedance Matrix :-

Consider a node basis matrix equation representing the power sm.

$$Y_{bus} V = I \qquad \text{(4}^{th}\text{ eq}^{n}\text{)}.$$

$$\therefore \quad V = Y_{bus}^{-1} I$$

$$Y_{bus}^{-1} = Z_{bus}$$

$$V = Z_{bus} I \quad \text{---------- (1)}$$

Modification of an Existing Bus impedance Matrix :

(Method 1) (or) Building algorithm of Z_{Bus} matrix :

* Let us denote the original Z_{bus} of a s/m with n-number of independant buses as Z_{orig}.
* when a branch of Z_b is added to Z_{orig}, it gets modified.
* It can be done in 4 different ways.

Case (i) :

Adding a branch of impedance Z_b from a new-bus-p to the reference bus.

Case (ii)

Adding a branch of impedance Z_b from a new bus-p to an existing bus-q.

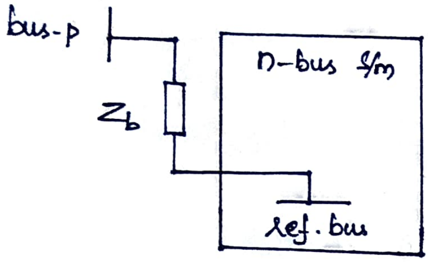
Case (iii)

Adding a branch of impedance Z_b from an existing bus-q to the reference bus.

Case (iv)

Adding a branch of impedance Z_b b/w two existing buses p and q.

Case(i) Adding Z_b from new bus-p to reference bus:

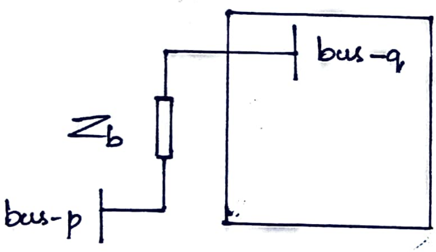


* Adding bus-p thro' Z_b to reference bus.

$$Z_{bus,new} = \left[\begin{array}{c|c} Z_{orig} & \begin{array}{c} 0 \\ \vdots \\ 0 \end{array} \\ \hline 0 \;\; 0 \cdots 0 & Z_b \end{array}\right]$$

* Addition will increase the order of Z_{bus} matrix by 1.
* $(n+1)^{th}$ column & row elements are '0' except the diagonal element.
* The diagonal element is Z_b.

Case(ii) Adding Z_b from new bus-p to existing bus-q

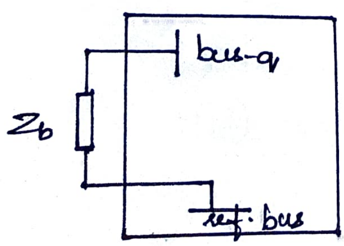


* Adding bus-p thro' Z_b to existing bus.

$$Z_{bus,new} = \left[\begin{array}{c|c} Z_{orig} & \begin{array}{c} Z_{1q} \\ Z_{2q} \\ \vdots \\ Z_{nq} \end{array} \\ \hline Z_{q1} \; Z_{q2} \cdots Z_{qn} & Z_{qq} + Z_b \end{array}\right]$$

* $(n+1)^{th}$ column & row elements are elements of q^{th} column & row.
* The diagonal element is $Z_{qq} + Z_b$

Case (iii) Adding Z_b from existing bus-q to reference bus.



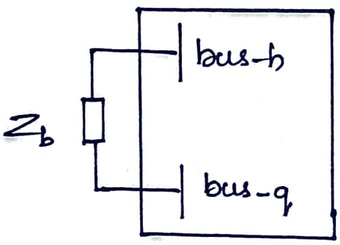
* Adding Z_b b/w existing & reference bus.

$$Z_{jk,act} = Z_{jk} - \frac{Z_{j(n+1)} Z_{(n+1)k}}{Z_{(n+1)(n+1)}}$$

* Perform Case (ii)
* Eliminate the extra node by node elimination techniqu

$$Z_{jk,act} = Z_{jk} - \frac{Z_{j(n+1)} Z_{(n+1)k}}{Z_{(n+1)(n+1)}}.$$

Case (v) Adding Z_b b/w two existing buses p and q :-



$$Z_{bus,new} = \left[\begin{array}{c|c} Z_{orig} & \begin{array}{c} Z_{1p}-Z_{1q} \\ Z_{2p}-Z_{2q} \\ \vdots \\ Z_{np}-Z_{nq} \end{array} \\ \hline Z_{p1}-Z_{q1} \quad Z_{p2}-Z_{q2} \cdots Z_{pn}-Z_{qn} & Z_{(n+1)(n+1)} \end{array}\right]$$

$$Z_{(n+1)(n+1)} = Z_b + Z_{pp} + Z_{qq} - 2Z_{pq}.$$

* Now the new bus becomes of order $(n+1)$.
* But it don't involves any new bus addition.

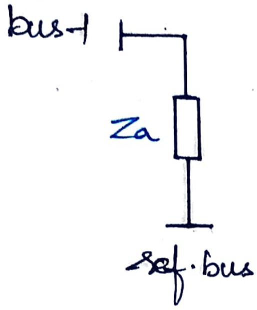
* Therefore the impedance matrix has to be reduced to n×n by eliminating $(n+1)^{th}$ column & row.

* This reduced bus impedance matrix is the actual new bus impedance matrix.

$$Z_{jk,act} = Z_{jk} - \frac{Z_{j(n+1)} Z_{(n+1)k}}{Z_{(n+1)(n+1)}}$$

Method 2:-

step 1: Consider an impedance Z_a connected from bus-1 to reference bus. Now Z_{bus} matrix will be,

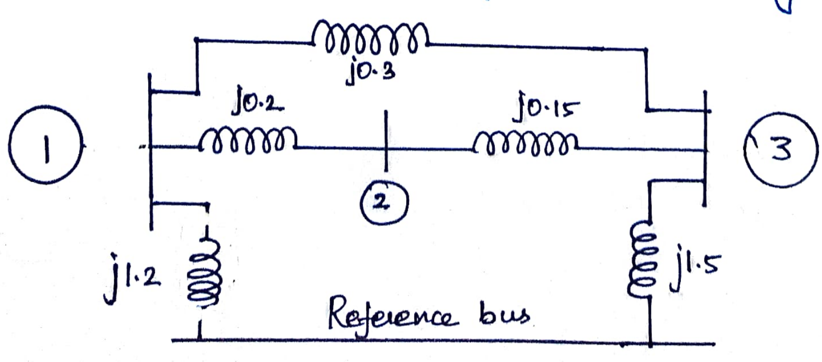


$$Z_{bus} = [Z_a]$$

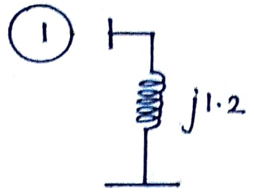
step 2: Add each element of impedance or reactance diagram one by one & modify Z_{bus} in each step, using any 4 cases discussed above.

Problems : 1

1. Determine Z_{bus} for system whose reactance diagram is shown. Preserve all the nodes. The impedances are given in p.u.

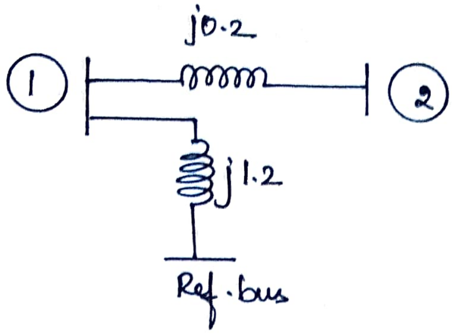


Step : 1



$$Z_{bus} = [j1.2]$$

Step : 2



* Adding bus-2 to existing bus-1 (p)
(Case ii)

In case (ii), $Z_{bus,new} = \begin{bmatrix} j1.2 & j1.2 \\ j1.2 & j1.4 \end{bmatrix}$

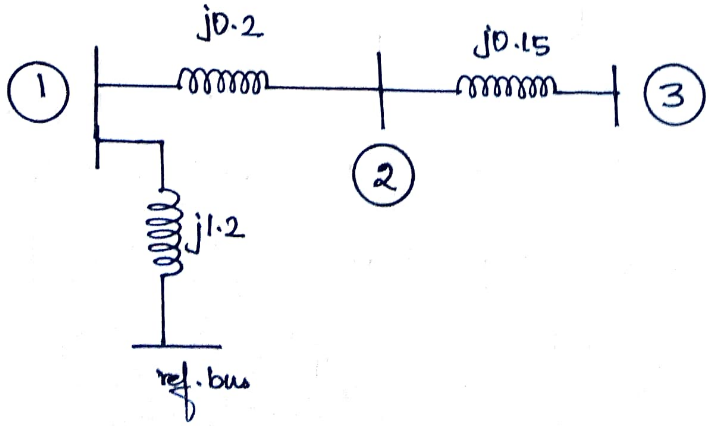
diagonal elt. $= Z_{qq} + Z_{b1}$

$= Z_{11} + j0.2$

$= j1.2 + j0.2$

$= j1.4$

Step : 3



* Adding bus-3 to existing bus-2, ie. case (ii) Modification

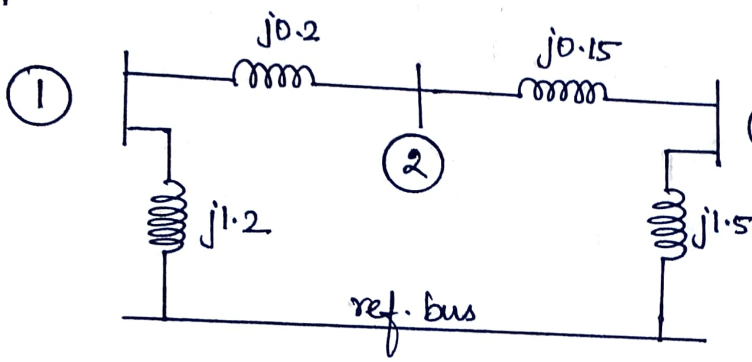
diagonal element $= Z_b + Z_{22}$

$= j0.15 + j1.4$

$= j1.55$

$$Z_{bus,new} = \begin{bmatrix} j1.2 & j1.2 & j1.2 \\ j1.2 & j1.4 & j1.4 \\ j1.2 & j1.4 & j1.55 \end{bmatrix}$$

Step :4



* Adding impedance $j1.5$ from existing bus – 3 to reference bus (case iii)

⇒ $Z_{bus,new}$ is formed by case (ii)

⇒ Last-row & column are eliminated.

$$Z_{bus,new} = \begin{bmatrix} j1.2 & j1.2 & j1.2 & j1.2 \\ j1.2 & j1.4 & j1.4 & j1.4 \\ j1.2 & j1.4 & j1.55 & j1.55 \\ j1.2 & j1.4 & j1.55 & j3.05 \end{bmatrix}$$

diag. elt $= Z_{qq} + Z_b$

$= j1.55 + j1.5$

$= j3.05$

Last-row & Column are eliminated, (4th column & 4th row).

$$Z_{jk,act} = Z_{jk} - \frac{Z_{j(n+1)} Z_{(n+1)k}}{Z_{(n+1)(n+1)}} \quad ; \quad n = 3, \quad k = 1,2,3, \quad j = 1,2,3$$

$$Z_{11,act} = Z_{11} - \frac{Z_{14}Z_{41}}{Z_{44}} = j1.2 - \frac{j1.2 \times j1.2}{j3.05} = j0.728$$

$$Z_{12,act} = Z_{12} - \frac{Z_{14}Z_{42}}{Z_{44}} = j1.2 - \frac{j1.2 \times j1.4}{j3.05} = j0.649$$

$$Z_{13,act} = Z_{13} - \frac{Z_{14}Z_{43}}{Z_{44}} = j1.2 - \frac{j1.2 \times j1.55}{j3.05} = j0.590.$$

$$Z_{21,act} = Z_{21} - \frac{Z_{24}Z_{41}}{Z_{44}} = j0.649.$$

$$Z_{22,act} = Z_{22} - \frac{Z_{24}Z_{42}}{Z_{44}} = j1.4 - \frac{j1.4 \times j1.4}{j3.05} = j0.757.$$

$$Z_{23,act} = Z_{23} - \frac{Z_{24}Z_{43}}{Z_{44}} = j1.4 - \frac{j1.4 \times j1.55}{j3.05} = j0.689.$$

$$Z_{31,act} = Z_{13,act} = j0.590.$$

$$Z_{32,act} = Z_{23,act} = j0.689$$

$$Z_{33,act} = Z_{33} - \frac{Z_{34}Z_{43}}{Z_{44}} = j1.55 - \frac{j1.55 \times j1.55}{j3.05} = j0.762.$$

$$Z_{bus,act} = \begin{bmatrix} j0.728 & j0.649 & j0.59 \\ j0.649 & j0.757 & j0.689 \\ j0.59 & j0.689 & j0.762 \end{bmatrix}$$

Step : 5

Adding impedance j0.3 b/w ① & ③. Case (iv).

$(n+1)^{th}$ column : difference b/w ① & ③ column.

$(n+1)^{th}$ row : difference b/w ① & ③ row.

$$Z_{(n+1)(n+1)} = Z_{44} = Z_b + Z_{bb} + Z_{qq} - 2Z_{bq}$$

$$= j0.3 + Z_{11} + Z_{33} - 2Z_{13}$$

$$= j0.3 + j0.728 + j0.762 - 2(j0.59).$$

$$= j0.61.$$

$$Z_{bus,new} = \begin{bmatrix} j0.728 & j0.649 & j0.59 & 0.138j \\ j0.649 & j0.757 & j0.689 & -0.04j \\ j0.59 & j0.689 & j0.762 & -0.172j \\ 0.138j & -0.04j & -0.172j & j0.61 \end{bmatrix}$$

Since this modification doesnot add a new bus, the 4th row & column has to be eliminated.

$$Z_{jk,act} = Z_{jk} - \frac{Z_{j(n+1)} Z_{(n+1)j}}{Z_{(n+1)(n+1)}}. \quad n = 3, \quad j = 1,2,3 \quad k = 1,2,3.$$

$$Z_{11,act} = Z_{11} - \frac{Z_{14} Z_{41}}{Z_{44}} = j0.728 - \frac{j0.138 \times j0.138}{j0.61} = j0.697.$$

$$Z_{12,act} = Z_{12} - \frac{Z_{14} Z_{42}}{Z_{44}} = j0.649 - \frac{j0.138 \times -0.04j}{j0.61} = j0.658.$$

$$Z_{13,act} = Z_{13} - \frac{Z_{14} Z_{43}}{Z_{44}} = j0.59 - \frac{j0.138 \times -j0.172}{j0.61} = j0.629.$$

$$Z_{21,act} = Z_{12,act} = j0.658.$$

$$Z_{22,act} = Z_{22} - \frac{Z_{24} Z_{42}}{Z_{44}} = j0.757 - \frac{(-j0.04)(-j0.04)}{j0.61} = j0.754.$$

$$Z_{23,act} = Z_{23} - \frac{Z_{24} Z_{43}}{Z_{44}} = j0.689 - \frac{(-0.04j)(-0.172j)}{j0.61} = j0.678$$

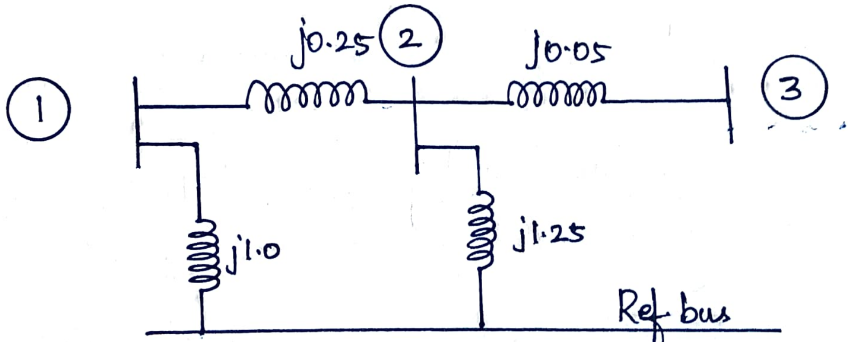
$Z_{31,act} = Z_{13,act} = j0.629.$

$Z_{32,act} = Z_{23,act} = j0.678.$

$$Z_{33,act} = Z_{33} - \frac{Z_{34}Z_{43}}{Z_{44}} = j0.762 - \frac{(j0.172)(-j0.172)}{j0.61} = j0.714$$

$$Z_{bus} = \begin{bmatrix} j0.697 & j0.658 & j0.629 \\ j0.658 & j0.754 & j0.678 \\ j0.629 & j0.678 & j0.714 \end{bmatrix}$$

2. Find the bus impedance (Z_{bus}) matrix for the s/m shown. whose reactance diagram is shown in fig. All the impedances are in p.u.



Step : 1

Consider the branch with impedance j1 p.u connected b/w bus-1 and reference.

$$Z_{bus} = [j1.0]$$

Step : 2

Connect bus-2 to bus-1 through an impedance j0.2

ie case (ii)

$$Z_{bus} = \begin{bmatrix} j1.0 & j1.0 \\ j1.0 & j1.0+j0.25 \end{bmatrix}$$

$$= \begin{bmatrix} j1.0 & j1.0 \\ j1.0 & j1.25 \end{bmatrix}$$

Step : 3 Connect impedance j1.25 from bus-2 to reference bus.

$$Z_{bus} = \begin{bmatrix} j1.0 & j1.0 & j1.0 \\ j1.0 & j1.25 & j1.25 \\ j1.0 & j1.25 & j1.25+j1.25 \end{bmatrix} = \begin{bmatrix} j1.0 & j1.0 & j1.0 \\ j1.0 & j1.25 & j1.25 \\ j1.0 & j1.25 & j2.5 \end{bmatrix}$$

Eliminating 3rd row 3rd column,

$$Z_{jk,act} = Z_{jk} - \frac{Z_{j(n+1)} Z_{(n+1)k}}{Z_{(n+1)(n+1)}}$$

$$Z_{11,act} = Z_{11} - \frac{Z_{13}Z_{31}}{Z_{33}} = j1.0 - \frac{j1.0 \times j1.0}{j2.5} = j0.6$$

$$Z_{12,act} = Z_{12} - \frac{Z_{13}Z_{32}}{Z_{33}} = j1.0 - \frac{j1.0 \times j1.25}{j2.5} = j0.5.$$

$$Z_{21,act} = Z_{12,act} = j0.5$$

$$Z_{22,act} = Z_{22} - \frac{Z_{23}Z_{32}}{Z_{33}} = j1.25 - \frac{j1.25 \times j1.25}{j2.5} = j0.625.$$

$$Z_{bus} = \begin{bmatrix} j0.6 & j0.5 \\ j0.5 & j0.625 \end{bmatrix}$$

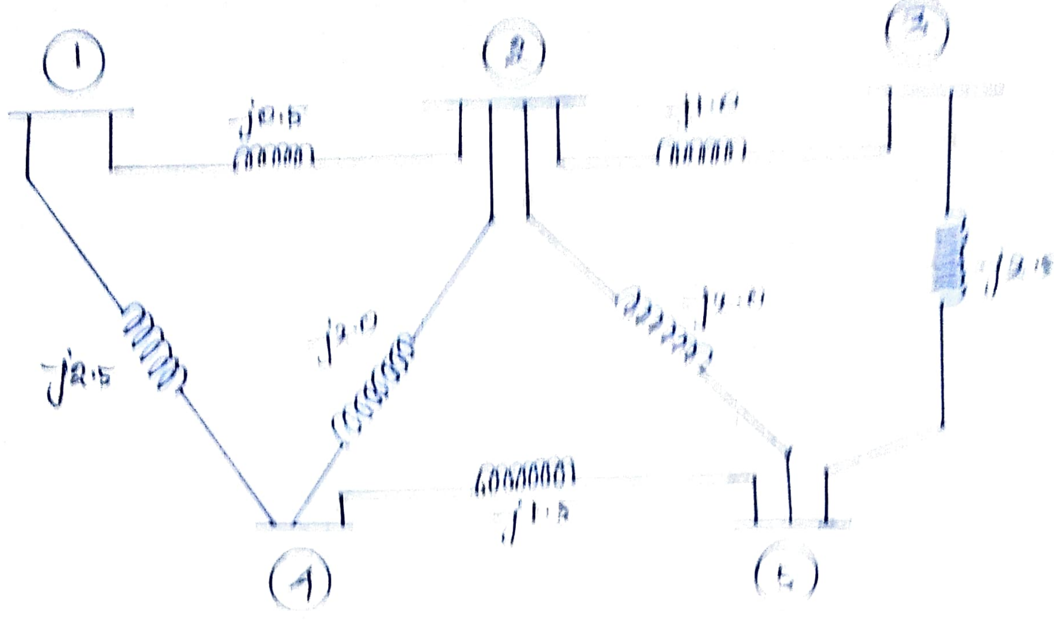
Step : 4

Connect the bus-3 to bus-2 through an impedance j0.05

$$Z_{bus} = \begin{bmatrix} j0.6 & j0.5 & j0.5 \\ j0.5 & j0.625 & j0.625 \\ j0.5 & j0.625 & j0.625+j0.05 \end{bmatrix}$$

$$Z_{bus} = \begin{bmatrix} j0.6 & j0.5 & j0.5 \\ j0.5 & j0.625 & j0.625 \\ j0.5 & j0.625 & j0.675 \end{bmatrix}$$

4. Bus Admittance Matrix : (Node elimination technique)



$$Y_{bus} = \begin{bmatrix} -j2.5-j0.5 & j0.5 & 0 & j2.5 & 0 \\ j0.5 & -j0.5-j2.0-j2.0-j1.0 & j1.0 & j2.0 & j2.0 \\ 0 & j1.0 & -j1.0-j2.5 & 0 & j2.5 \\ j2.5 & j2.0 & 0 & -j2.5-j2-j1.5 & j1.5 \\ 0 & j2.0 & j2.5 & j1.5 & -j1.5-j2.0-j2.5 \end{bmatrix}$$

$$Y_{bus} = \begin{bmatrix} -3j & j0.5 & 0 & j2.5 & 0 \\ j0.5 & -j5.5 & j1.0 & j2.0 & j2.0 \\ 0 & j1.0 & -j3.5 & 0 & j2.5 \\ j2.5 & j2.0 & 0 & -j6.0 & j1.5 \\ 0 & j2.0 & j2.5 & j1.5 & -j6.0 \end{bmatrix}$$

Eliminating node 5:

$$Y_{jk,new} = Y_{jk} - \frac{Y_{jn}Y_{nk}}{Y_{nn}}.$$

$j = 1, 2, 3, 4.$

$k = 1, 2, 3, 4.$

$n = 5.$

$$Y_{11,new} = Y_{11} - \frac{Y_{15}Y_{51}}{Y_{55}} = -j3 - \frac{0 \times 0}{-j6} = -j3.$$

$$Y_{12,new} = Y_{12} - \frac{Y_{15}Y_{52}}{Y_{55}} = j0.5 - \frac{0 \times j2.0}{-j6} = j0.5$$

$$Y_{13,new} = Y_{13} - \frac{Y_{15}Y_{35}}{Y_{55}} = 0 - \frac{0 \times j2.5}{-j6} = 0.$$

$$Y_{14,new} = Y_{14} - \frac{Y_{15}Y_{54}}{Y_{55}} = j2.5 - \frac{0 \times j1.5}{-j6} = j2.5$$

$$Y_{21,new} = Y_{21} - \frac{Y_{25}Y_{51}}{Y_{55}} = j0.5 - \frac{j2.0 \times 0}{-j6} = j0.5$$

$$Y_{22,new} = Y_{22} - \frac{Y_{25}Y_{52}}{Y_{55}} = -j5.5 - \frac{j2.0 \times j2.0}{-j6} = -j4.8333.$$

$$Y_{23,new} = Y_{23} - \frac{Y_{25}Y_{53}}{Y_{55}} = j1.0 - \frac{j2.0 \times j2.5}{-j6} = j1.833.$$

$$Y_{24,new} = Y_{24} - \frac{Y_{25}Y_{54}}{Y_{55}} = j2.0 - \frac{j2.0 \times j1.5}{-j6} = 2.5j$$

$$Y_{31,new} = Y_{31} - \frac{Y_{35}Y_{51}}{Y_{55}} = 0 - \frac{j2.5 \times 0}{-j6} = 0$$

$$Y_{32,new} = Y_{32} - \frac{Y_{35}Y_{52}}{Y_{55}} = j1.0 - \frac{j2.5 \times j2.0}{-j6} = 1.833j.$$

$$Y_{33,new} = Y_{33} - \frac{Y_{35}Y_{53}}{Y_{55}} = -j3.5 - \frac{j2.5 \times j2.5}{-j6} = -2.45j.$$

$$Y_{34,new} = Y_{34} - \frac{Y_{35}Y_{54}}{Y_{55}} = 0 - \frac{j2.5 \times j1.5}{-j6} = 0.625j.$$

$$Y_{41,new} = Y_{41} - \frac{Y_{45}Y_{51}}{Y_{55}} = j2.5 - \frac{j1.5 \times 0}{-j6} = j2.5.$$

$$Y_{42,new} = Y_{42} - \frac{Y_{45}Y_{52}}{Y_{55}} = j2.0 - \frac{j1.5 \times j2.0}{-j6} = 2.5j.$$

$$Y_{43,new} = Y_{43} - \frac{Y_{45}Y_{53}}{Y_{55}} = 0 - \frac{j1.5 \times j2.5}{-j6} = 0.625j.$$

$$Y_{44,new} = Y_{44} - \frac{Y_{45}Y_{54}}{Y_{55}} = -j6 - \frac{j1.5 \times j1.5}{-j6} = -5.625j.$$

$$Y_{bus,new} = \begin{bmatrix} -j3 & j0.5 & 0 & j2.5 \\ j0.5 & -j4.8333 & j1.833 & 2.5j \\ 0 & 1.833j & -2.45j & 0.625j \\ 2.5j & 2.5j & 0.625j & -5.625j \end{bmatrix}$$

Formation of Y_{bus} by Inspection or Two rule method:

Bus: The meeting pt. of various components in a power S/m is called Bus. (or) [element used to connect one component to another]. [∵ conductor made of Cu or Al]

* Bus admittance Y and bus impedance Z matrices are two important network matrices which finds application in many power S/m planning & operational studies.

* Y_{bus} applied in load flow & optimal load flow analysis. & Z-bus matrix used in SC Analysis

$$[Y][V] = [I]$$

$$[Z][I] = [V].$$

$V = IR$.
$V = IZ$
$\frac{V}{Z} = I$
$\frac{1}{Z} = Y$.
$VY = I$.

* These matrices are important building blocks of power s/m modelling & analysis.

*

$$\begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}.$$

* Y_{ii} (i.e. $Y_{11}, Y_{22}, \ldots$) is equal to sum of admittances of all elements connected to the i^{th} node.

* Y_{ij} ($Y_{12}, Y_{21} \ldots$) is equal to negative of the sum of the admittances of all elements connected b/w the nodes i & j.

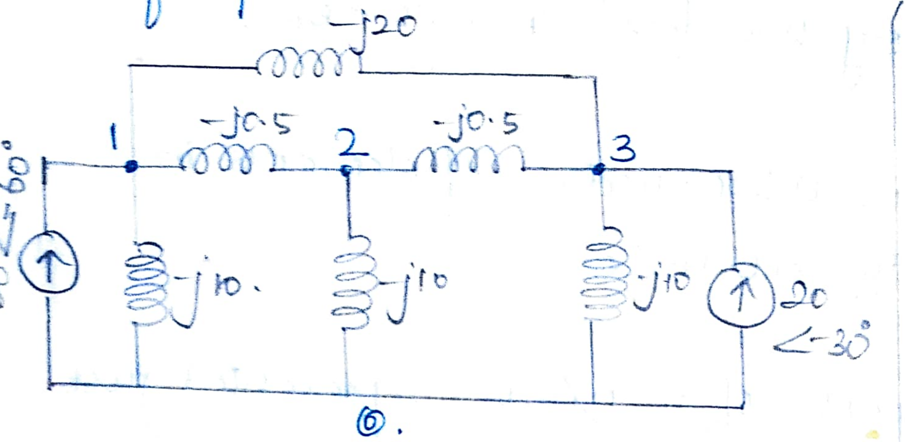
* $Y_{ij} = 0$ if there is no line b/w i & j.

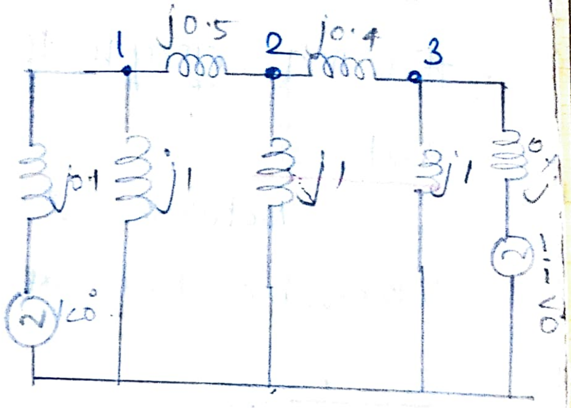
<u>Uses</u> :

1. Y-bus is used in solving load flow problems.
2. Simplicity in data preparation.
3. It can be easily formed & modified for any changes in the network.
4. It reduces computer memory & time requirement because of <u>sparse matrix</u> ?

Each bus is connected to only a few nearby buses. so many off-diagonal elts are zero.

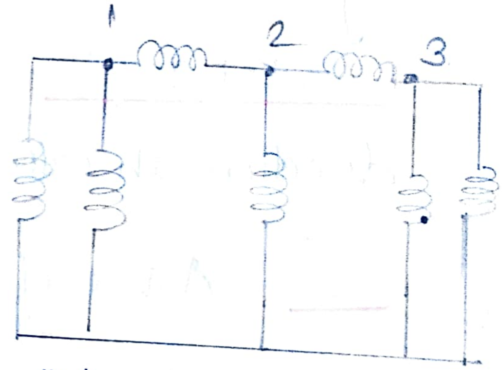
1. For the n/w shown in Fig. write the elements of Y-bus mat by inspection method.





* I source open ckt.ed + V source short ckt ed.

* I source open ckt.
V Source short ckt.



$$Y_{bus} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix}$$

$$= \begin{bmatrix} -j0.5-j10-j20 & j0.5 & j20 \\ j0.5 & -j10-j0.5-j0.5 & j0.5 \\ j20 & j0.5 & -j10-j20-j0.5 \end{bmatrix}$$

$$Y_{bus} = \begin{bmatrix} -j30.5 & j0.5 & j20 \\ j0.5 & -j11 & j0.5 \\ j20 & j0.5 & -j30.5 \end{bmatrix}$$

$$Y_{bus} = \begin{bmatrix} \frac{1}{j0.1}+\frac{1}{j1}+\frac{1}{0.5} & -\frac{1}{j0.5} & 0 \\ -\frac{1}{j0.5} & \frac{1}{j.5}+\frac{1}{j1}+\frac{1}{j.4} & -\frac{1}{0.4} \\ 0 & -\frac{1}{j0.4} & \frac{1}{j.4}+\frac{1}{j}+\frac{1}{j} \end{bmatrix}$$

$$= \begin{bmatrix} -j13 & j2 & j0 \\ j2 & j5.5 & j2.5 \\ j0 & j2.5 & j13.5 \end{bmatrix}$$

Formation of Y_{bus} & Z_{bus} using Singular Transformation.

* Using graph theory we find Y_{bus} & Z_{bus}.

Network:-

Network is an interconnection of elements in various branches at different nodes.

Graph:-

If each element of a n/w is represented just by a line, then its called graph.

Oriented graph:- If each element is assigned a direction its called oriented graph. (+ve dir^n. of I).

Tree:- A tree of a graph is a part of that graph which has sufficient no. of branches to connect all the nodes without forming a closed path.

No. of tree branches = No. of nodes − 1.

Links: The elements of graph which are not included in the tree are known as links. They form a subgraph, called cotree.

links = no. of elts − no. of tree branches.

UNIT – II

POWER FLOW ANALYSIS

Importance of Power Flow Analysis:

Power flow studies (analysis) are conducted to investigate the following features of a power system network.

* Bus voltage profile to acceptable value and also system voltage profile.
* The effect of temporary loss of transmission capacity mainly for security studies.
* System loss minimization and improvement of voltage regulation.
* The line flows.
* The effect of change in configuration and incorporating new circuits on system loading.
* Economic system operation.
* Transformer tap setting for economic operation.
* Change of conductor sizes and system voltages for improvements to an existing system.
* Transient or dynamic stability analysis.

Purpose of power flow Study:

The power flow analysis is the steady state analysis during normal operating conditions.

The following were computed...

* Voltage magnitude & phase angles at the bus.
* Line flows in MW and MVAr supplied by generators.
* overloaded conditions, poor Voltages.
* Line losses in MW.
* Angle seperation between any two buses gives an idea f[...] small disturbance stability.
* optimal load distribution.
* Impact of any change in generation and transmission on the system loading.
* choice of approximate rating and tap changed of the p[...] transformer in the system.
* Assessment of the influence of any change in conductor size and system Voltage level.

Types of Buses:

* Buses are meeting points of Various components.
* Each Bus in a power system is associated with four quantities, they are real, reactive power, magnitude of Voltage and phase angle of Voltage.
* In load flow problem, 2 quantities are specified for each bus & remaining two quantities are obtained by solving the load flow equations.
* 3 types of Buses.

i) Load Bus (PQ Bus)

ii) Generator Bus (or) Voltage controlled Bus (or) PV Bus.

iii) Slack Bus (or) Swing Bus (or) Reference Bus.

Bus type	Quantities specified	Quantities to be obtained
Load Bus	P, Q	$\lvert V \rvert, \delta$
Generator Bus	P, $\lvert V \rvert$	Q, δ
Slack Bus	$\lvert V \rvert, \delta$	P, Q.

$P = P_G - P_D$

$Q = Q_G - Q_D$

$\lvert V \rvert$: magnitude of bus Voltage.

δ : phase of bus Voltage.

P_G, Q_G : Real and Reactive powers generated by generators to the bus respectively.

P_D, Q_D : Real and Reactive powers drawn by the loads connected to the bus respectively.

Load Bus: P, Q.

(when the real and reactive components of power are specified for the bus, then it is called Load bus. The load flow equations can be solved to find the magnitude and phase of bus voltage.

In a load bus, the voltage is allowed to vary within permissible limits, for eg. ±5%

Generator Bus:

(when the real power and magnitude of bus voltage are specified then the bus is called generator bus).

The load flow equation can be solved to find the reactive power and phase of bus voltage.

Usually for generator buses, reactive power limits will be specified.

Slack Bus:

The bus is called slack Bus, if the magnitude & phase of bus voltage are specified for the bus.

It is the reference bus for load flow solution & usually one of the generator bus is selected as slack bus.

Need for Slack Bus:

Basically the power system has only two types of buses and they are load and generator buses. In these buses only power injected by generators and power drawn by loads are specified but power loss in transmission lines are not accounted.

In a power system the total power generated will be equal to sum of power consumed by loads & losses.

Sum of complex of generator = Sum of complex power of loads + Total power loss in transmission lines.

* The transmission line losses can be estimated only if the real and reactive power of all buses are known.

* The power in the buses will be known only after solving the load flow equations.

* For these reasons, the real & reactive power of one of the generator bus is not specified and its called slack bus.

* It is assumed that the slack bus generates the real & reactive power required for transmission line losses.

* Hence for a slack bus, the magnitude & phase of bus voltage are specified, and real & reactive powers are obtained through the load flow solution.

Formulation of Load flow Equations

The node basis matrix equation of n-bus s/m is,

$$Y_{bus} V = I \quad \text{--- (1)}$$

Y_{bus}: $(n \times n)$, V: $(n \times 1)$, I: $(n \times 1)$

$$\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_p \\ \vdots \\ I_n \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1p} & \cdots & Y_{1n} \\ Y_{21} & Y_{22} & \cdots & Y_{2p} & \cdots & Y_{2n} \\ \vdots \\ Y_{p1} & Y_{p2} & \cdots & Y_{pp} & \cdots & Y_{pn} \\ Y_{n1} & Y_{n2} & \cdots & Y_{np} & \cdots & Y_{nn} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_p \\ \vdots \\ V_n \end{bmatrix} \quad \text{--- (2)}$$

I_p : current injected to bus-p.

V_p : Voltage of bus-p.

'I_p' can be written as,

$$I_p = Y_{p1}V_1 + Y_{p2}V_2 + \cdots Y_{pp}V_p + \cdots Y_{pn}V_n \quad \text{——— (3)}$$

$$\therefore I_p = \sum_{q=1}^{p-1} Y_{pq}V_q + Y_{pp}V_p + \sum_{q=p+1}^{n} Y_{pq}V_q \quad \text{——— (4).}$$

$$S_p = P_p + jQ_p.$$

S_p : Complex power of bus-p. I_p^* : Conjugate of I_p.

P_p : Real power of bus-p.

Q_p : Reactive power of bus-p.

WKT, $S_p = V_p I_p^*$

$$V_p I_p^* = P_p + jQ_p.$$

Taking conjugate on both sides,

$$(V_p I_p^*)^* = (P_p + jQ_p)^*$$

$$V_p^* I_p = P_p - jQ_p.$$

$$\boxed{I_p = \frac{P_p - jQ_p}{V_p^*}} \quad \text{——— (5)}$$

Equate (4) & (5).

$$\frac{P_p - jQ_p}{V_p^*} = \sum_{q=1}^{p-1} Y_{pq}V_q + Y_{pp}V_p + \sum_{p+1}^{n} Y_{pq}V_q \quad \text{——— (6)}$$

<u>Method 1: Gauss – Seidal Method:</u>

Eqn (6) can be written as,

$$Y_{pp}V_p = \frac{P_p - jQ_p}{V_p^*} - \sum_{q=1}^{p-1} Y_{pq}V_q - \sum_{p+1}^{n} Y_{pq}V_q$$

$$V_p = \frac{1}{Y_{pp}}\left[\frac{P_p - jQ_p}{V_p^*} - \sum_{q=1}^{p-1} Y_{pq}V_q - \sum_{q=p+1}^{n} Y_{pq}V_q\right] \quad \text{——— (7)}$$

i.e.,

$$V_p = \frac{1}{Y_{pp}}\left[\frac{P_p - jQ_p}{V_p^*} - \sum_{\substack{q=1 \\ q\neq p}}^{n} Y_{pq}V_q\right] \quad \text{——— (8).}$$

The above equation is the load flow equation for Gauss-seidal method.

Newton-Raphson Method:

Equation (6) can be written as,

$$P_p - jQ_p = V_p^* \sum_{q=1}^{n} Y_{pq}V_q \quad \text{——— (9)}$$

$V_p = e_p + jf_p$; e_p, f_p : real & imaginary part of V_p ——— (9.1)

$V_q = e_q + jf_q$; e_q, f_q : real & imaginary part of V_q.

$Y_{pq} = G_{pq} - jB_{pq}$; G_{pq}, B_{pq} – Conductance & susceptance of the admittance Y_{pq} respectively.

Substitute in eq^n^ (9).

$$P_p - jQ_p = (e_p + jf_p)^* \sum_{q=1}^{n} (G_{pq} - jB_{pq})(e_q + jf_q)$$

$$= e_p - jf_p \sum_{q=1}^{n} (G_{pq}e_q + jG_{pq}f_q - jB_{pq}e_q - (-1)B_{pq}f_q)$$

$$= e_p - jf_p \sum_{q=1}^{n} \left[(G_{pq}e_q + B_{pq}f_q) + j(G_{pq}f_q - B_{pq}e_q)\right]$$

$$= \sum_{q=1}^{n} (e_p - jf_p)\left[(G_{pq}e_q + B_{pq}f_q) + j(G_{pq}f_q - B_{pq}e_q)\right]$$

$$= \sum_{q=1}^{n} \Big[e_p(e_q G_{pq} + f_q B_{pq}) + j e_p (f_q G_{pq} - e_q B_{pq}) - j f_p (e_q G_{pq} + f_q B_{pq}) - j^2 f_p (f_q G_{pq} - e_q B_{pq}) \Big].$$

$$= \sum_{q=1}^{n} \Big[e_p(e_q G_{pq} + f_q B_{pq}) + f_p (f_q G_{pq} - e_q B_{pq}) + j e_p (f_q G_{pq} - e_q B_{pq}) - j f_p (e_q G_{pq} + f_q B_{pq}) \Big].$$

$$P_p - jQ_p = \sum_{q=1}^{n} \Big[e_p(e_q G_{pq} + f_q B_{pq}) + f_p (f_q G_{pq} - e_q B_{pq}) \Big] - j \sum_{q=1}^{n} \Big[f_p (e_q G_{pq} + f_q B_{pq}) - e_p (f_q G_{pq} - e_q B_{pq}) \Big] \quad (10)$$

On seperating the real & imaginary parts of above eqn.,

$\therefore$

$$P_p = \sum_{q=1}^{n} \Big[e_p(e_q G_{pq} + f_q B_{pq}) + f_p (f_q G_{pq} - e_q B_{pq}) \Big] \quad (11)$$

$$Q_p = \sum_{q=1}^{n} \Big[f_p(e_q G_{pq} + f_q B_{pq}) - e_p (f_q G_{pq} - e_q B_{pq}) \Big]. \quad (12)$$

From equation (9.1)

$$|V_p|^2 = e_p^2 + f_p^2 \quad (13).$$

Eqn.s (11), (12), (13) are called load flow equations of Newton- Raphson method.

Load flow Solution By Gauss - Seidal Method :-

The Gauss - seidal method is an iterative algorithm for solving a set of non-linear load flow equations.

The non-linear load flow equation is g^n by,

$$V_p = \frac{1}{Y_{pp}}\left[\frac{P_p - jQ_p}{V_p^*} - \sum_{q=1}^{p-1} Y_{pq}V_q - \sum_{q=p+1}^{n} Y_{pq}V_q\right] \quad (13.1)$$

$P = 1, 2, 3 \ldots n.$

* The node voltages $V_1, V_2, V_3 \ldots V_n$ can be obtained by from above equation for $P = 1, 2, 3 \ldots n$.
* In Gauss - seidal method, initial values of voltages are assumed & denoted as $V_1^0, V_2^0, V_3^0 \ldots V_n$.
* substitute these initial values in above eq^n. and by taking $P = 1$, the revised value of bus-1 voltage V_1' is computed.
* The revised value of bus voltage V_1' is replaced for initial value V_1^0 & revised bus-2 voltage V_2' is computed.
* Replace V_1' for V_1^0 and V_2' for V_2^0 & perform the calculation for bus-3. & so on.

Modified Equation, for iteration

$$V_p^{k+1} = \frac{1}{Y_{pp}}\left[\frac{P_p - jQ_p}{(V_p^k)^*} - \sum_{q=1}^{p-1} Y_{pq}V_q^{k+1} - \sum_{q=p+1}^{n} Y_{pq}V_q^k\right] \quad (14)$$

V_i^k = k^{th} iteration value of bus voltage V_i.

V_i^{k+1} = $(k+1)^{th}$ iteration value of bus voltage V_i.

* In above eqn to compute $(k+1)^{th}$ iteration value of bus-p V_q the $(k+1)^{th}$ iteration values of voltages are used for all buses less than p. &

k^{th} iteration values of voltages are used for all buses greater than or equal to p.

* <u>To estimate the reactive power</u> from the bus voltages & admittances.

$$\frac{P_p - jQ_p}{V_p^*} = \sum_{q=1}^{p-1} Y_{pq} V_q + Y_{pp} V_p + \sum_{q=p+1}^{n} Y_{pq} V_q .$$

$$P_p - jQ_p = V_p^* \left[\sum_{q=1}^{p-1} Y_{pq} V_q + \sum_{p=q}^{n} Y_{pq} V_q \right] \text{——— (15)}$$

During $(k+1)^{th}$ iteration,

$$P_p^{k+1} - jQ_p^{k+1} = (V_p^k)^* \left[\sum_{q=1}^{p-1} Y_{pq} V_q^{k+1} + \sum_{q=p}^{n} Y_{pq} V_q^k \right] \text{——— (16)}$$

The reactive power of bus-p during $(k+1)^{th}$ iteration is given by imaginary part of above eqn.

Reactive power of bus-p during $(k+1)^{th}$ iteration $\Big\}$ $Q_p^{k+1} = (-1) \, \mathrm{Im} \left\{ (V_p^k)^* \left[\sum_{q=1}^{p-1} Y_{pq} V_q^{k+1} + \sum_{q=p}^{n} Y_{pq} V_q^k \right] \right\}$ ——— (17).

* For each iteration, the reactive power of generator bus is calculated using eqn (17) & checked with specified limits.

* If it violates the specified limits then the reactive power of the bus is equated to limit violated & treated as

load bus. If it doesnot violates the limits then the bus is treated as generator bus.

Computation of Slack Bus power and line flows:

The slack bus power can be calculated after computing the bus voltage upto the specified accuracy.

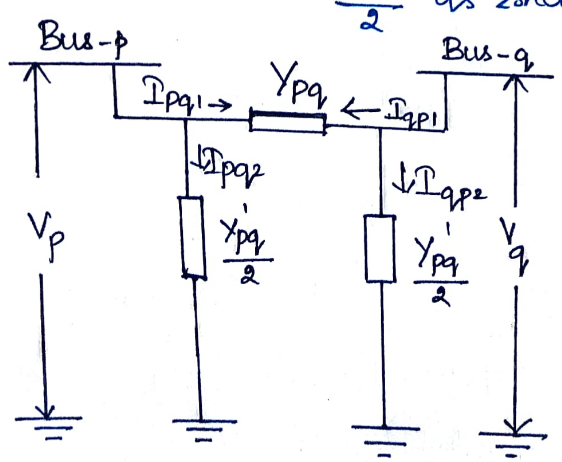
Eqn. (15) can be used to calculate slack bus power. Here bus-p is a slack bus.

$$P_p - jQ_p = V_p^* \sum_{q=1}^{n} Y_{pq} V_q^k \quad \text{—— (18).}$$

The line flows are the power fed by the buses into various lines & they are calculated as shown below.

Consider a line connecting bus-p and bus-q. Usually the transmission line is connected to buses using transformers at its ends.

The π-equivalent of the transmission line with transformers at its end consist of a series admittance Y_{pq} and shunt admittances $\frac{Y'_{pq}}{2}$ as shown.



$$I_{pq} = I_{pq1} + I_{pq2}$$

$$= (V_p - V_q) Y_{pq} + V_p \frac{Y'_{pq}}{2}$$

Complex power injected by bus-p in line pq } $S_{pq} = P_{pq} - jQ_{pq} = V_p^* I_{pq}$

$$= V_p^* \left[(V_p - V_q) Y_{pq} + V_p \frac{Y'_{pq}}{2} \right]$$

$$I_{qp} = I_{qp1} + I_{qp2} = (V_p - V_q) Y_{pq} + V_q \frac{Y'_{pq}}{2} .$$

Complex power injected by bus-q in line-pq } $S_{qp} = P_{qp} - jQ_{qp} = V_q^* I_{qp}$

$$= V_q^* \left[(V_q - V_p) Y_{pq} + V_q \frac{Y'_{pq}}{2} \right]$$

Power loss in the txn. line-pq } $\Rightarrow S_{pq,loss} = S_{pq} + S_{qp}$.

Procedure for load flow solution by Gauss-Seidal Method:

Step: 1 Assume a flat voltage profile $1+j0$ for all buses except the slack bus. The voltage of slack bus is the specified voltage & it is not modified in any iteration.

Step: 2 Assume a suitable value of ϵ called convergence criterion. Here ϵ is a specified change in bus voltage that is used to compare the actual change in bus voltage b/w k^{th} & $(k+1)^{th}$ iteration.

Step : 3 Set iteration count $K=0$ & assumed voltage profile of the buses are denoted as $V_1^0, V_2^0, V_3^0 \ldots V_n^0$ except slack bus.

Step : 4 Set bus count, $P=1$

Step : 5 Check for slack bus. If it is a slack bus then go to step 12 otherwise go to next step.

Step : 6 Check for generator bus. If it is a generator bus go to next step, or else if it is a load bus goto step-9

Step : 7 Temporarily set $|V_p^k| = |V_p|_{spec}$ and phase of V_p^k as the k^{th} iteration value if the bus-p is the generator bus where $|V_p|_{spec}$ is the specified magnitude of voltage for bus-p. Then calculate the reactive power of the generator bus using the following equation.

$$Q_{p,cal}^{k+1} = (-1) \, Im \left\{ (V_p^k)^* \left[\sum_{q=1}^{p-1} Y_{pq} V_q^{k+1} + \sum_{q=p}^{n} Y_{pq} V_q^{k} \right] \right\}$$

Im : Imaginary part of.

⇒ If the calculated 'Q' is within specified limits then its generator bus. and set $Q_p = Q_{p,cal}^{k+1}$ for this iteration and go to step-8.

⇒ If Q violates specified limit, then treat it as load bus.

$Q_{p,cal}^{k+1} < Q_{p,min}$ then $Q_p = Q_{p,min}$

$Q_{p,cal}^{k+1} > Q_{p,max}$, then $Q_p = Q_{p,max}$.

→ For load bus, take V_p^k for $(k+1)$ th iteration value. then goto step-9.

Step:8 For generator bus the magnitude of voltage doesn't change and so for all iterations the magnitude of bus voltage is the specified value.

Phase of bus voltage:

$$V_{p,temp}^{k+1} = \frac{1}{Y_{pp}}\left[\frac{P_p - jQ_p}{(V_p^k)^*} - \sum_{q=1}^{p-1} Y_{pq} V_q^{k+1} - \sum_{q=p+1}^{n} Y_{pq} V_q^k\right]$$

$$\delta_p^{k+1} = \tan^{-1}\left[\frac{\text{Im. part of } V_{p,temp}^{k+1}}{\text{Real part of } V_{p,temp}^{k+1}}\right].$$

Now, the $(k+1)^{th}$ iteration voltage of the generator bus is given by

$$V_p^{k+1} = |V_p|_{spec} \angle \delta_p^{k+1}.$$

Step:9 For the load bus the $(k+1)^{th}$ iteration value of load bus voltage, V_p^{k+1} can be calculated using the following equation:

$$V_p^{k+1} = \frac{1}{Y_{pp}}\left[\frac{P_p - jQ_p}{(V_p^k)^*} - \sum_{q=1}^{p-1} Y_{pq} V_q^{k+1} - \sum_{q=p+1}^{n} Y_{pq} V_q^k\right]$$

Step :10 An acceleration factor, α can be used for faster convergence. If acceleration factor is specified then modify the $(k+1)^{th}$ iteration value of bus-p voltage using the following equation.

$$V_{p,acc}^{k+1} = V_p^k + \alpha (V_p^{k+1} - V_p^k).$$

Then set, $V_p^{k+1} = V_{p,acc}^{k+1}$.

Mostly $\alpha = 1.6$.

Step :11

Calculate the change in bus-p voltage using the relation.

$$\Delta V_p^{k+1} = V_p^{k+1} - V_p^k.$$

Step :12 Calculate the steps 5 to 11 until all the bus voltages have been calculated. For this increment the bus count by 1 and go to step. 5 until the bus count is n.

Note :

Pure load bus P, Q ⇒ −ve.

Gen. Bus P, Q ⇒ +ve.

1. The system data for a load flow solution are gn. in table 1 & 2. Determine the Voltages at the end of first iteration by Gauss-Seidal Method. Take $\alpha = 1.6$.

Line admittance:

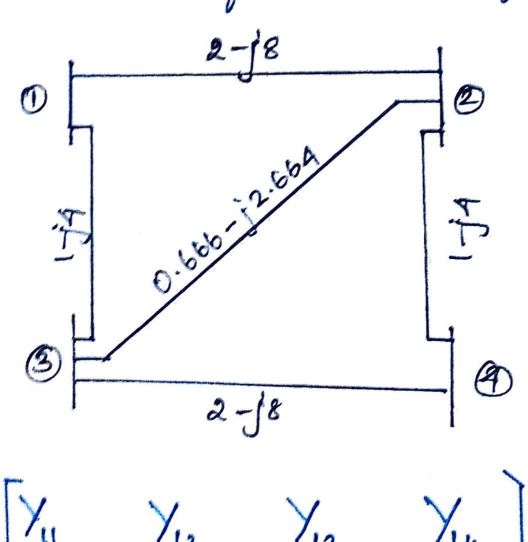
Buscode	Admittance
1-2	2-j8
1-3	1-j4
2-3	0.666-j2.664
2-4	1-j4
3-4	2-j8

Bus Specifications

Buscode	P	Q	V	Remarks
1	-	-	$1.06\angle 0°$	Slack
2	0.5	0.2	-	PQ
3	0.4	0.3	-	PQ
4	0.3	0.1	-	PQ.

Solution:

From 1st table, the Single line diagram can be drawn as



* $$Y_{bus} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix}$$

$Y_{11} = 2-j8+1-j4 = 3-j12.$

$Y_{22} = 2-j8+0.666-j2.664+1-j4 = 3.666-j14.664.$

$Y_{33} = 1-j4+0.666-j2.664+2-j8 = 3.666-j14.664$

$Y_{44} = 1-j4+2-j8 = 3-j12$

$Y_{12} = Y_{21} = -(2-j8) = -2+8j.$

$Y_{13} = Y_{31} = -(1-j4) = -1+j4.$

$Y_{14} = Y_{41} = 0.$

$Y_{23} = Y_{32} = -(0.666-j2.664) = -0.666+j2.664.$

$Y_{24} = Y_{42} = -(1-j4) = -1+j4.$

$Y_{34} = Y_{43} = -(2-j8) = -2+j8.$

$$Y_{bus} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix}$$

$$Y_{bus} = \begin{bmatrix} 3-j12 & -2+8j & -1+j4 & 0 \\ -2+8j & 3.666-j14.664 & -0.666+j2.664 & -1+j4 \\ -1+j4 & -0.666+j2.664 & 3.666-j14.664 & -2+j8 \\ 0 & -1+j4 & -2+j8 & 3-j12 \end{bmatrix}$$

step:1

The initial values of bus voltages are considered as 1p.u except the slack bus.

$V_2^0 = 1+j0$; $V_3^0 = 1+j0$; $V_4^0 = 1+j0$.

The bus-1 is the slack bus, its voltage value is given & remain at the specified value for all iterations.

i.e. $V_1^0 = V_1^1 = \dots V_1^k$; $V_1 = 1.06+j0$ p.u.

From eqn. (13.1)

The $(k+1)^{th}$ iteration value of a PQ (load bus) - p is given by,

$$V_p^{k+1} = \frac{1}{Y_{pp}}\left[\frac{P_p - jQ_p}{(V_p^k)^*} - \sum_{q=1}^{p-1} Y_{pq} V_q^{k+1} - \sum_{q=p+1}^{n} Y_{pq} V_q^k\right]$$

<u>Iteration : 1</u>

$k=0$,

$P = 1,2,3,4$ (no. of bus = 4)

$V_1^1 = V_1^0 = 1.06+j0$ p.u (Bus-1 is a slack bus)

$$V_2^1 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{(V_2^0)^*} - \sum_{q=1}^{1} Y_{21} V_1^1 - \sum_{q=3}^{4} Y_{23} V_3^0 - Y_{24} V_4^0\right]$$

$$= \frac{1}{3.666 - j14.664}\left[\frac{-0.5+j0.2}{1-j0} - (-2+j8)(1.06+j0) - (-0.666+j2.664)(1+j0) - (-1+j4)(1+j0)\right]$$

$$= \frac{-0.5+j0.2+2.12-j8.48+0.666-j2.664+1-j4}{3.666-j14.664}.$$

$$= \frac{3.486 - j14.944}{3.666 - j14.664}$$

$$V_2^1 = 1.0119 - j0.0290 \text{ p.u.}$$

$\alpha = 1.6,$

$$V_{2,acc}^1 = V_2^0 + \alpha(V_2^1 - V_2^0)$$

$$= 1+j0 + 1.6(1.0119 - j0.0290 - 1+j0).$$

$$= 1.0190 - j0.0464.$$

Now, $V_2^1 = V_{2,acc}^1 = 1.0190 - j0.0464$ pu

$K=1,\ P=3$

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{(V_3^0)^*} - Y_{31}V_1^1 - Y_{32}V_2^1 - Y_{34}V_4^0\right].$$

$$= \frac{1}{3.666 - j14.664}\left[\frac{-0.4+j0.3}{1-j0} - (-1+j4)(1.06) - (0.666+j2.664)(1.0190 - j0.0464) - (-2+j8)(1+j0)\right]$$

$$= \frac{-0.4 + j0.3 + 1.06 - j4.24 - (-0.555 + j2.7455) + 2 - j8}{3.666 - j14.664}$$

$$V_3^1 = 0.9942 - j0.0293 \text{ p.u.}$$

$$V_{3,acc}^1 = V_3^0 + \alpha(V_3^1 - V_3^0)$$

$$= 1 + 1.6(0.9942 - j0.0293 - 1).$$

$$V_3^1 = V_{3,acc}^1 = 0.9907 - j0.0469.$$

$K=0,\ P=4$

$$V_4^1 = \frac{1}{Y_{44}}\left[\frac{P_4 - jQ_4}{(V_4^0)^*} - Y_{41}V_1^1 - Y_{42}V_2^1 - Y_{43}V_3^1\right]$$

$$= \frac{1}{3-j12}\left[\frac{-0.3+j0.1}{1-j0} - (0\times 1.06) - (-1+j4)(1.019-j0.0464) - (-2+j8)(0.9907-j0.0469)\right]$$

$$= \frac{-0.3+j0.1 - (-0.8334+j4.1224) - (-1.6062+j8.0194)}{3-j12}$$

$$= 2.13 - j0.0683 \text{ p.u.}$$

$$V_{4,acc}^{1} = V_4^{0} + \alpha (V_4^{1} - V_4^{0})$$

$$= 1 + 1.6(0.9864 - j0.0683 - 1)$$

$$V_4^{1} = V_{4,acc}^{1} = 0.9782 - j0.1093$$

Answer:

The bus voltages at the end of first iteration are,

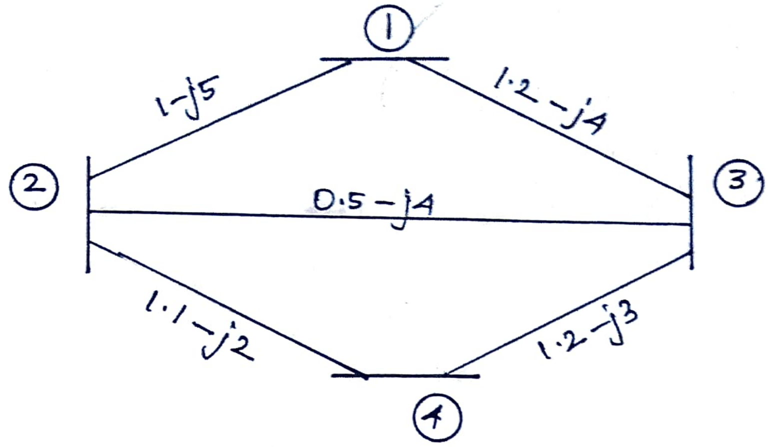
$$V_1^{1} = 1.06 + j0$$

$$V_2^{1} = 1.019 - j0.0464$$

$$V_3^{1} = 0.9907 - j0.0469$$

$$V_4 = 0.9782 - j0.1093$$

2 For the system shown in fig. determine the voltages at the end of first iteration by Gauss-Seidal Method. Take $\alpha = 1$ and bus specifications are gr.



Bus specifications.

Buscode	P	Q	V	Remarks
1	-	-	$1.06\angle 0^\circ$	Slack
2	0.5	$0.1 \le Q_2 \le 1$	1.04	PV
3	0.4	0.3	-	PQ
4	0.2	0.1	-	PQ

$$Y_{bus} = \begin{bmatrix} 2.2-j9 & -1+j5 & -1.2+j4 & 0 \\ -1+j5 & 2.6-j11 & -0.5+j4 & -1.1+j2 \\ -1.2+j4 & -0.5+j4 & 2.9-j11 & -1.2+j3 \\ 0 & -1.1+j2 & -1.2+j3 & 2.3-j5 \end{bmatrix}$$

$V_1^0 = V_1^1 = \dots V_1^K = V_1 = 1.06 + j0$ p.u (Slack bus)

$V_2^0 = 1.04 + j0$ p.u (Generator Bus)

$V_3^0 = 1 + j0$ p.u (Load Bus)

$V_4^0 = 1 + j0$ p.u (Load Bus)

Generator bus (positive) ; Load bus (negative).

The calculations of bus voltages for first iteration,

$V_1^1 = V_1^0 = 1.06 + j0$ p.u (Bus 1 is a slack Bus)

The bus-2 is a generator bus and so calculate its reacti power, Q_2.

$$Q_{p,cal}^{K+1} = -1 \times Im\left\{ (V_p^K)^* \left[\sum_{q=1}^{p-1} Y_{pq} V_q^{K+1} + \sum_{q=p}^{n} Y_{pq} V_q^K \right] \right\}$$

$P = 2, k = 0, n = 4.$

$$Q_{2,cal}^1 = -1 \times Im\left\{ (V_2^0)^* \left[Y_{21} V_1^1 + Y_{22} V_2^0 + Y_{23} V_3^0 + Y_{24} V_4^0 \right] \right\}.$$

$$Q_{2,cal}^1 = -1 \times Im\left\{ (1.04 \left[(-1+5j)(1.06+0j) + (2.6-j11)(1.04+j0) + (-0.5+j4)(1+j0) + (-1.1+j2)(1+j0) \right] \right\}$$

$$= -1 \times Im\left\{ 1.04 \left[-1.06 + 5.3j + 2.704 - 11.44j + (-0.5+4j) + (-1.1+2j) \right] \right\}$$

$$= -1 \times Im\left\{ 1.04 \left[0.044 - 0.14j \right] \right\}.$$

$$= -1 \times Im\left\{ 0.04576 - 0.1456j \right\}$$

$$= 0.1456 \text{ p.u.}$$

The specified value for Q_2 is $0.1 \leq Q_2 \leq 1.0$.

The value is within range, so its not violated. Theref its treated as generator bus.

$P_2 = 0.5, \quad Q_2 = 0.1456, \quad V_2^0 = 1.04 + j0$

Since its generator bus, the $|V_2^1| = |V_2|_{spec}$ and phase of V_2^1 is given by the phase of $V_{2,temp}^1$.

$$V_{p,temp}^{k+1} = \frac{1}{Y_{pp}}\left[\frac{P_p - jQ_p}{(V_p^k)^*} - \sum_{q=1}^{p-1} Y_{pq} V_q^{k+1} - \sum_{q=p+1}^{n} Y_{pq} V_q^k\right].$$

$P = 2, \; k = 0, \; n = 4.$

$$V_{2,temp}^1 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{(V_2^0)^*} - Y_{21} V_1^1 - Y_{23} V_3^0 - Y_{24} V_4^0\right]$$

$$= \frac{1}{2.6 - j11}\left[\frac{0.5 - j0.1456}{1.04 - j0} - (-1+j5)(1.06+j0) - (-0.5+j4)(1+j0) - (-1.1+j2)(1+j0)\right].$$

$$= \frac{1}{2.6 - j11}\left[0.4804 - j0.14 + 1.06 - j5.3 + 0.5 - j4 + 1.1 - j2\right].$$

$$= \frac{3.1408 - j11.44}{2.6 - j11} \qquad = 1.0488 + 0.0376j.$$

$$= 2.05^\circ.$$

$$= \frac{11.8633\angle -74.65^\circ}{11.3031\angle -76.70^\circ}$$

$$= 1.0496\angle 2.05^\circ \text{ p.u.}$$

$$\delta_2^1 = \angle V_{2,temp}^1 = \tan^{-1}\left[\frac{\text{Imaginary part of } V_{p,temp}}{\text{Real part of } V_{p,temp}}\right]$$

$$= 2.05^\circ.$$

$$V_2^1 = |V_2|_{spec} \angle \delta_2^1$$

$$= 1.04 \angle 2.05^{\circ}$$

$$= 1.0393 + j0.0372 \text{ p.u.}$$

The bus-3 and bus 4 are load buses. The voltages of load bu
are calculated using the following equation.

$$V_p^{K+1} = \frac{1}{Y_{pp}}\left[\frac{P_p - jQ_p}{(V_p^k)^*} - \sum_{q=1}^{p-1} Y_{pq} V_q^{K+1} - \sum_{q=p+1}^{n} Y_{pq} V_q^k\right].$$

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{(V_3^0)^*} - Y_{31}V_1^1 - Y_{32}V_2^1 - Y_{34}V_4^0\right].$$

$$= \frac{1}{2.9 - j11}\left[\frac{-0.4 + j0.3}{1 - j0} - (-1.2 + j4)(1.06 + j0) - (-0.5 + j4)(1.0393 + j0.0372) - (-1.2 + j3)(1 + j0)\right].$$

$$= \frac{1}{2.9 - j11}\left[-0.4 + j0.3 + 1.272 - j4.24 - (-0.6685 + j4.13 + (1.2 - j3)\right]$$

$$= \frac{2.7405 - j11.0786}{2.9 - j11}$$

$$= 1.0031 - j0.0154 \text{ p.u}$$

$$V_4^1 = \frac{1}{Y_{44}}\left[\frac{P_4 - Q_4}{(V_4^0)^*} - Y_{41}V_1^1 - Y_{42}V_2^1 - Y_{43}V_3^1\right]$$

$$= \frac{1}{2.3 - j5}\left[\frac{-0.2 + j0.1}{1 - j0} - (0 \times 1.06) - (-1.1 + j2)(1.0393 + j0.0 - (-1.2 + j3)(1.0031 - j0.0154)\right]$$

$$= \frac{2.1751 - j4.9655}{2.3 - j5}$$

$$= 0.9848 - j0.0179 \text{ p.u}$$

The bus voltages at the end of first iteration are,

$V_1^1 = 1.06 + j0$

$V_2^1 = 1.0393 + j0.0371$

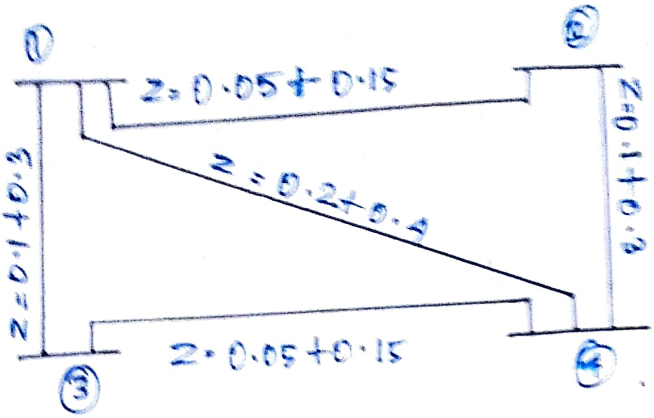
$V_3^1 = 1.0031 - j0.0154.$

$V_4^1 = 0.9848 - j0.0179.$

3. In the s/m shown in figure. Determine the bus voltages at the end of first Gauss - Seidal Iteration.

Buscode	P	Q	V	Remarks
1	-	-	$1.05\angle 0^\circ$	slack bus
2	0.5	$0.3 \le Q_2 \le 1.0$	1.07	PV
3	-1.0	0.5	-	PQ
4	0.3	-0.1	-	PQ

Line	R (p.u)	X (p.u)
1-2	0.05	0.15
1-3	0.10	0.30
1-4	0.20	0.40
2-4	0.10	0.30
3-4	0.05	0.15



$Y_{12} = \frac{1}{0.05+0.15j} = 2-j6$ $Y_{11} = Y_{12}+Y_{13}+Y_{14} = 4-j11$

$Y_{13} = \frac{1}{0.1+j0.3} = 1-j3$ $Y_{22} = Y_{12}+Y_{24} = 3-j9.$

$Y_{14} = \frac{1}{0.2+j0.4} = 1-j2$ $Y_{33} = Y_{13}+Y_{34} = 3-j9$

$Y_{24} = \frac{1}{0.1+j0.3} = 1-j3$ $Y_{44} = 4-j11$

$Y_{34} = \frac{1}{0.05+j0.15} = 2-j6$

$$Y_{bus} = \begin{bmatrix} 4-j11 & -2+j6 & -1+j3 & -1+j2 \\ -2+j6 & 3-j9 & 0 & -1+j3 \\ -1+j3 & 0 & 3-j9 & -2+j6 \\ -1+j2 & -1+j3 & -2+j6 & 4-j11 \end{bmatrix}$$

The initial Value of load bus Voltages are.

$V_1^1 = V_1^0 = 1.05+j0$ p.u $(P=1)$.

$V_2^0 = 1.07+j0$

$V_3^0 = 1+j0$

$V_4^0 = 1+j0.$

$K = 0,\ P = 2,$

Bus - 2 is a generator bus, so reactive power should be calculated.

$$Q_{p,cal}^{k+1} = (-1)\, Im\left\{ (V_p^k)^* \left[\sum_{q=1}^{p-1} Y_{pq} V_q^{k+1} + \sum_{q=p}^{n} Y_{pq} V_q^{k} \right] \right\}$$

$$Q_{2,cal}^{1} = (-1) Im\left\{ (V_2^0)^* \left[Y_{21}V_1^1 + Y_{22}V_2^0 + Y_{23}V_3^0 + Y_{24}V_4^0 \right] \right\}$$

$$= (-1) Im\left\{ (1.07 - j0) \left[(-2 + j6)(1.05 + j0) + (3 - j9)(1.07 + j0) + (0 \times V_3^0) + (-1 + j3)(1 + j0) \right] \right\}$$

$$= 0.3531 \text{ p.u.}$$

Q limits : $0.3 \leq Q_2 \leq 1.0$

* The calculated Q value is within limit, so its treated as generator bus itself. $Q_2 = Q_{p,cal}^{k+1} = Q_{2,cal}^{1} = 0.3531$ p.u.

* So voltage is 1.07 p.u only.

* Now to calculate phase of bus - 2. (P = 2).

$$V_{p,temp}^{k+1} = \frac{1}{Y_{pp}} \left[\frac{P_p - jQ_p}{(V_p^k)^*} - \sum_{q=1}^{p-1} Y_{pq} V_q^{k+1} - \sum_{q=p+1}^{n} Y_{pq} V_q^{k} \right].$$

$$= \frac{1}{Y_{22}} \left[\frac{P_2 - jQ_2}{(V_2^0)^*} - Y_{21}V_1^1 - Y_{23}V_3^0 - Y_{24}V_4^0 \right].$$

$$= \frac{1}{3 - j9} \left\{ \frac{0.5 - j0.3531}{1.07 - j0} - (-2 + j6)(1.05 + j0) - (0 \times V_3^0) \right.$$

$$= 1.0825 \angle 1.9^\circ.$$

$$\delta_p^{k+1} = \tan^{-1}\left[\frac{\text{Im. part of } V_{p,temp}^{k+1}}{\text{Real part of } V_{p,temp}^{k+1}}\right]$$

$$= 1.9^\circ.$$

The voltage of bus-3 & bus-4 are calculated using the followi equation.

$$V_p^{k+1} = \frac{1}{Y_{pp}}\left[\frac{P_p - jQ_p}{(V_p^k)^*} - \sum_{q=1}^{p-1} Y_{pq} V_q^{k+1} - \sum_{q=p+1}^{n} Y_{pq} V_q^k\right].$$

$$V_3' = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{(V_3^0)^*} - Y_{31}V_1' - Y_{32}V_2' - Y_{34}V_4^0\right].$$

$$= \frac{1}{3-j9}\left[\frac{-1-j0.5}{1-j0} - (-1+j3)(1.05+j0) - (0\times V_2') - (-2+j6)(1+j0)\right].$$

$$= \frac{1}{3-j9}\left[-1-j0.5+j.05-j3.15+2-j6\right]$$

$$= 1.0399 \angle -6.44^\circ.$$

$$= 1.033 - j0.1166 \text{ p.u}$$

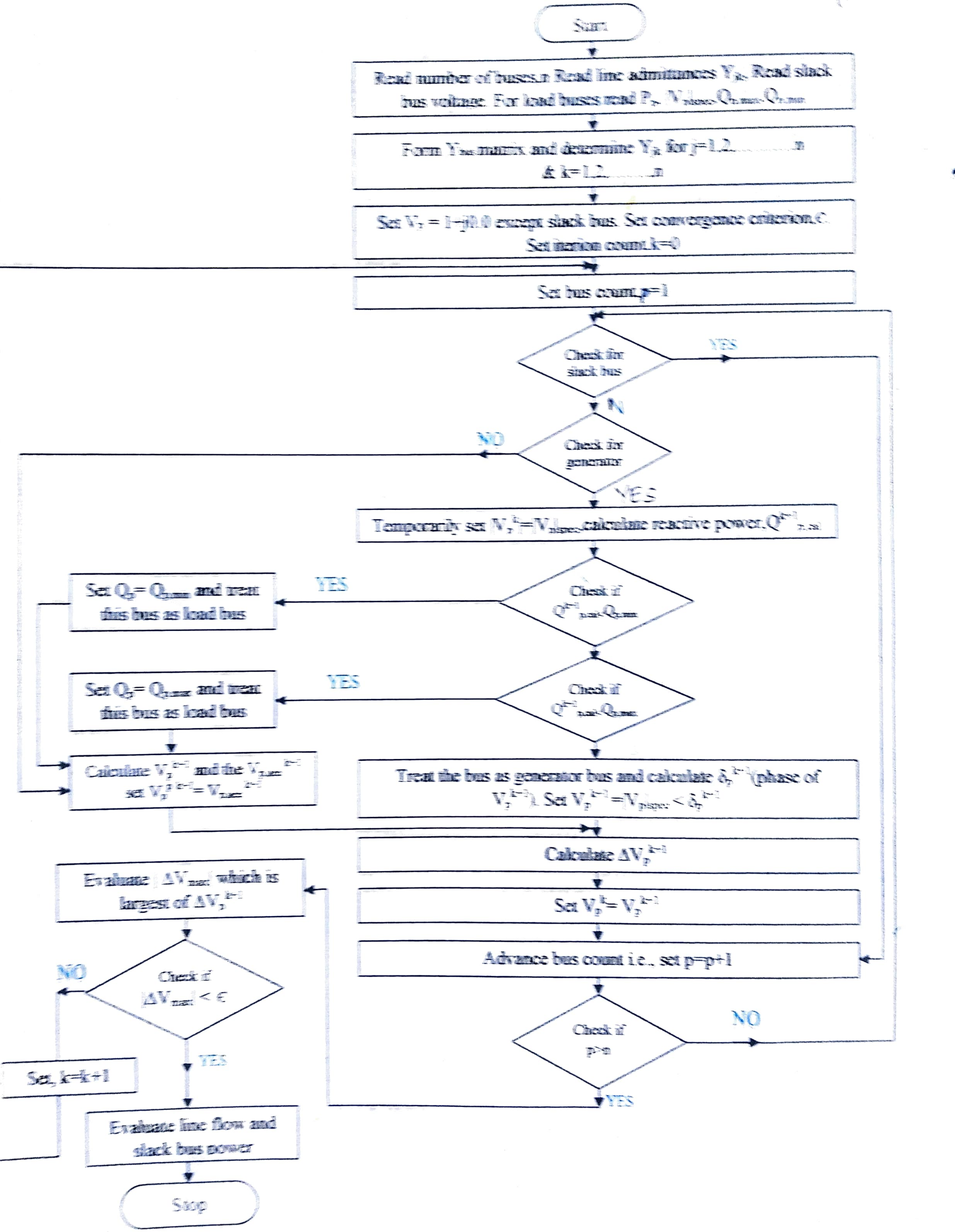
$$V_4' = 1/Y_{44}\left[\frac{P_4 - jQ_4}{(V_4^0)^*} - Y_{41}V_1' - Y_{42}V_2' - Y_{43}V_3'\right].$$

$$= \frac{1}{4-j11}\left[\frac{0.3+j0.1}{1-j0} - (-1+j2)(1.05+j0) - (-1+j3)(1.0694+j0 - (-2+j6)(1.033-j0.1166)\right].$$

$$= 1.0458 \angle -1.44^\circ.$$

$$V_4' = 1.0455 - j0.0263 \text{ p.u.}$$

Flowchart - Gauss Seidal



* The newton-Raphson method of load flow analysis is an iterative method which approximates the set of non-linear simultaneous equations to a set of linear simultaneous equations using Taylor's series expansion.

* Very reliable method.

* Fast in convergence compared to Gauss-seidal Method.

* Very popular for large system studies.

* The rate of convergence is relatively independent of system size.

* Both rectangular & polar co-ordinates can be used to find bus voltages.

* Large requirement of computer memory is one of its demerit.

Procedure for Newton-Raphson Method [Polar Form]

Step 1: Assume a flat voltage profile $1\angle 0°$ $(1+j0)$ for all load buses.

Step 2: Form Y_{bus} and convert it into polar form.

Step 3: Assume suitable ϵ, called convergence criterion.

Step 4: Set iteration count $k=0$.

Step 5: Set bus count, $P=1$.

Step 6: Check whether p^{th} bus is P-V bus (or) PQ bus or slack bus. If it is a slack bus increment the bus count $P=P+1$. then goto step 5. or go to next step.

Step 7: check for PV bus. If it is a PV bus go to next step, or go to step-11.

Step 8: Calculate real power P_p^{k+1} and ΔP_p^{k+1} using following equations, for all PV bus.

$$P_p^{k+1} = |V_p^k| \sum_{q=1}^{n} |Y_{pq}||V_q^k| \cos(\theta_{pq} + \delta_q^k - \delta_p^k)$$

$$\Delta P_p^{k+1} = P_{p,spec}^k - P_p^{k+1}.$$

Step 9: Calculate Q_p^{k+1} and ΔQ_p^{k+1} for all P-V bus.

$$Q_p^{k+1} = -|V_p^k| \sum_{q=1}^{n} |Y_{pq}||V_q^k| \sin(\theta_{pq} + \delta_q^k - \delta_p^k)$$

$$\Delta Q_p^{k+1} = Q_{p,spec}^k - Q_p^{k+1}.$$

Step 10: check for Q-limit.

⇒ If Q_p^{k+1} is within the limits, then treat it as generator bus.

⇒ If Q_p^{k+1} is violating the limits, then treat it as load bus.

* $Q_p^{k+1} < Q_{p,min} \Rightarrow Q_p^{k+1} = Q_{p,min}$.

* $Q_p^{k+1} > Q_{p,max} \Rightarrow Q_p^{k+1} = Q_{p,max}$.

Step 11: Calculate P_p^{k+1} & Q_p^{k+1}, ΔP_p^{k+1}, ΔQ_p^{k+1} for all PQ bu[s] using above said formulas.

Step 12: Check if $P < n$, if Yes go to step 5 and

Increment the bus count $p = p+1$. otherwise go to next step.

step 13: Calculate the Jacobian matrix elements using the following equations.

Elements of J_A

If $(p \neq q) = \frac{\partial P_p}{\partial \delta_q} = -|V_p^k||Y_{pq}||V_q^k| \sin(\theta_{pq} + \delta_q^k - \delta_p^k)$

If $(p = q) = \frac{\partial P_p}{\partial \delta_p} = \sum_{\substack{q=1 \\ \neq p}}^{n} |V_p||V_q||Y_{pq}| \sin(\theta_{pq} + \delta_q - \delta_p)$

Elements of J_B

If $(p \neq q) = \frac{\partial P_p}{\partial |V_q|} = |V_p||Y_{pq}| \cos(\theta_{pq} + \delta_q - \delta_p)$

If $(p = q) = \frac{\partial P_p}{\partial V_p} = 2|V_p||Y_{pp}| \cos(\theta_{pp}) + \sum_{\substack{q=1 \\ \neq p}}^{n} |V_q||Y_{pq}| \cos(\theta_{pq} + \delta_q - \delta_p)$

Elements of J_C :

If $(p \neq q) = \frac{\partial Q_p}{\partial \delta_q} = -|V_p||V_q||Y_{pq}| \cos(\theta_{pq} + \delta_q - \delta_p)$

If $(p = q) = \frac{\partial Q_p}{\partial \delta_p} = \sum_{\substack{q=1 \\ \neq p}}^{n} |V_p||V_q||Y_{pq}| \cos(\theta_{pq} + \delta_q - \delta_p)$

Elements of J_D :

If $(p \neq q) = \frac{\partial Q_p}{\partial |V_q|} = -|V_p||Y_{pq}| \sin(\theta_{pq} + \delta_q - \delta_p)$

If $(p = q) \Rightarrow \frac{\partial Q_p}{\partial |V_p|} = -2|V_p||Y_{pp}| \sin \theta_{pp} - \sum_{\substack{q=1 \\ \neq p}}^{n} |V_q||Y_{pq}| \sin(\theta_{pq} + \delta_q - \delta_p)$

Step 15 : Calculate $\Delta \delta$ and ΔV using following equation called as state correction vector.

$$\begin{bmatrix} \Delta \delta \\ \Delta V \end{bmatrix} = \begin{bmatrix} J_A & J_B \\ J_C & J_D \end{bmatrix}^{-1} \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}$$

Step 16 : Calculate

$$V_p^{k+1} = V_p^k + \Delta V_p^k \quad \text{for all PQ bus.}$$

$$\delta_p^{k+1} = \delta_p^k + \Delta \delta_p^k \quad \text{for all bus.}$$

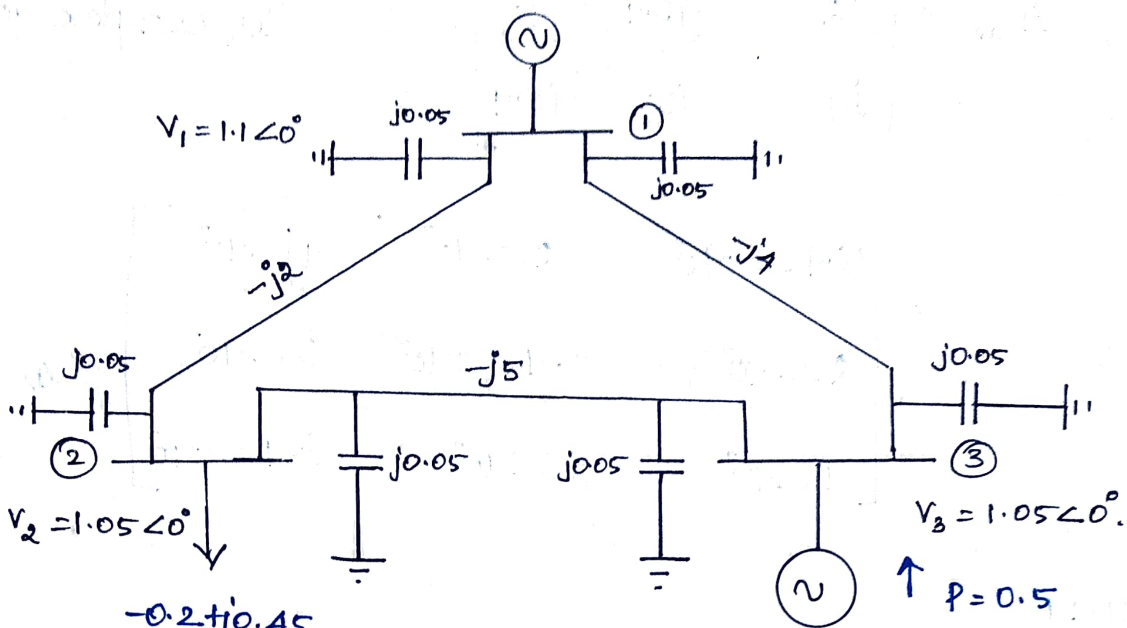
Step 17 : Check for Convergence. If condition satisfies, calculate line flows, otherwise go to step 4, incrementing $k = k+1$.

1. For the system shown in figure with bus 1 as slack bus. Obtain the power flow solution using Newton-Raphson method at the end of first iteration.

Bus	Impedance	Half line charging admittance
1-2	j0.5	j0.05
2-3	j0.2	j0.05
1-3	j0.25	j0.05

Bus code	Generation		Load		|V|	Type
	P_G	Q_G	P_L	Q_L		
1	-	-	-	-	1.1	Slack
2	-	-	-20	45	-	PQ
3	50	0	-	-	1.05	P-V

Complete Network will be as,



step :2

$$Y_{bus} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix} \quad [\text{total bus} = 3]$$

$$Y_{11} = \frac{1}{0.5j} + \frac{1}{0.25j} + j0.05 + 0.05j = -5.9j.$$

$$Y_{12} = \frac{1}{0.5j} = 2j$$

$$Y_{13} = \frac{1}{0.25j} = -4j.$$

$$Y_{21} = Y_{12} = -2j.$$

$$Y_{22} = j0.05 + 0.05j + \frac{1}{j0.5} + \frac{1}{0.2j} = -6.9j$$

$$Y_{23} = \frac{1}{j0.2} = -5j.$$

$$Y_{31} = Y_{13} = -4j.$$

$$Y_{32} = Y_{23} = -5j.$$

$$Y_{33} = j0.05 + j0.05 + \frac{1}{0.2j} + \frac{1}{0.25j} = -8.9j$$

$$Y_{bus} = \begin{bmatrix} -j5.9 & j2 & j4 \\ j2 & -j6.9 & j5 \\ j4 & j5 & -j8.9 \end{bmatrix}$$ in complex form.

$$= \begin{bmatrix} 5.9\angle -90^\circ & 2\angle 90^\circ & 4\angle 90^\circ \\ 2\angle 90^\circ & 9\angle -90^\circ & 5\angle 90^\circ \\ 4\angle 90^\circ & 5\angle 90^\circ & 8.9\angle -90^\circ \end{bmatrix}$$ in polar fo

step : 1

$V_1^0 = 1.1\angle 0^\circ. \Rightarrow |V_1^0| = 1.1$ & $\delta_1^0 = 0$

$V_2^0 = 1.0^\circ \angle 0^\circ. \Rightarrow |V_2^0| = 1.0$ & $\delta_2^0 = 0$.

$V_3^0 = 1.05\angle 0^\circ \Rightarrow |V_3^0| = 1.05$ & $\delta_3^0 = 0$.

step : 3

No need of ϵ, Convergence criterion as first iteration solution is only asked.

step 4 :

$k = 0$

Step 5

$P = 1$

step : 6

Short procedure for 6 – 12 steps.

Calculate P_p^{k+1}, Q_p^{k+1} for all bus except slack bus.

bus 1 ← slack bus.

bus 2, 3 ← PQ & PV bus.

so calculate real & reactive power for bus 2, 3.

Bus 2 : PQ Bus:

$p = 2; \; n = 3.$

$k = 0;$

$$P_p^{k+1} = |V_p^k| \sum_{q=1}^{n} |Y_{pq}||V_q^k| \cos(\theta_{pq} + \delta_q^k - \delta_p^k)$$

$$P_2^1 = |V_2^0| \left[|Y_{21}||V_1^0| \cos(\theta_{21} + \delta_1^0 - \delta_2^0) + |V_2^0||Y_{22}| \cos(\theta_{22} - \delta_2^0 - \delta_2^0) + |V_3^0||Y_{23}| \cos(\theta_{23} + \delta_3 - \delta_2) \right]$$

$$= 1.0 \left[2 \times 1.1 \times \cos 90^\circ + 1.0 \times 6.9 \times \cos(-90^\circ) + 1.05 \times 5 \times \cos 90^\circ \right]$$

$$P_2^1 = 0.$$

$$Q_p^{k+1} = -|V_p^k| \sum_{q=1}^{n} |Y_{pq}||V_q^k| \sin(\theta_{pq} + \delta_q^k - \delta_p^k)$$

$$Q_2^1 = -|V_2^0| \left[|Y_{21}||V_1| \sin(\theta_{21} + \delta_1^0 - \delta_2^0) + |V_2||Y_{22}| \sin(\theta_{22} + \delta_2^0 - \delta_2^0) + |V_3||Y_{23}| \sin(\theta_{23} + \delta_3^0 - \delta_2^0) \right]$$

$$= -1.0 \left[2 \times 1.1 \times \sin 90^\circ + 1.0 \times 6.9 \times \sin(-90^\circ) + 5 \times 1.05 \times \sin 90^\circ \right]$$

$$= -2.2 + 6.9 - 5.25$$

$$Q_2^1 = -0.55.$$

Bus : 2

$$P_2^1 = 0$$
$$Q_2^1 = -0.55$$

Bus 3 : PV bus, Calculate P_p^{k+1}, Q_p^{k+1}.

$P = 3$

$k = 0$

$n = 3$

$$P_p^{k+1} = |V_p^k| \sum_{q=1}^{n} |V_q^k||Y_{pq}| \cos(\theta_{pq} + \delta_q^k - \delta_p^k)$$

$$P_3^1 = |V_3^0| \Big[|V_1^0||Y_{31}| \cos(\theta_{31} + \delta_1^0 - \delta_3^0) + |V_2^0||Y_{32}| \cos(\theta_{32} + \delta_2^0 - \delta_3^0) + |V_3^0||Y_{33}| \cos(\theta_{33} + \delta_3^0 - \delta_3^0) \Big].$$

$$= 1.05 \Big[1.1 \times 4 \times \cos(90^\circ + 0 - 0) + 1 \times 5 \times \cos(90^\circ + 0^\circ - 0^\circ) + 1.05 \times 8.9 \times \cos(-90^\circ + 0^\circ - 0^\circ) \Big].$$

$$P_3^1 = 0.$$

$$Q^{k+1} = -|V_p^k| \sum_{q=1}^{n} |V_q^k||Y_{pq}| \sin(\theta_{pq} + \delta_q^k - \delta_p^k).$$

$$Q_3^1 = -|V_3^0| \Big[|V_1^0||Y_{31}| \sin(\theta_{31} + \delta_1^0 - \delta_3^0) + |V_2^0||Y_{32}| \sin(\theta_{32} + \delta_2^0 - \delta_3^0) + |V_3^0||Y_{33}| \sin(\theta_{33} + \delta_3^0 - \delta_3^0) \Big].$$

$$= -1.05 \Big[1.1 \times 4 \times \sin(90^\circ + 0 - 0) + 1.0 \times 5 \times 90^\circ - 1.05 \times 8.9 \times \sin(-90^\circ) \Big].$$

$$= 5.54 - 4.62 + 9.81255$$

$$= -0.05775$$

Bus : 3

$P_3' = 0$

$Q_3' = -0.05775$

Step : 13 Calculate Jacobian Matrix Elements.

The equation is,

$$\begin{bmatrix} \Delta\delta \\ \Delta V \end{bmatrix} = \begin{bmatrix} J_A & J_B \\ J_C & J_D \end{bmatrix}^{-1} \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} \quad \text{——— (1)}$$

* we have to calculate δ for all buses. (bus 2 & 3 in this prob) except slack.

so $\Delta\delta_2 , \Delta\delta_3$.

* we have to calculate ΔV for PQ bus. (bus no. 2 in this prob.)

i.e. ΔV_2

so eqn. no. (1) will be written as,

$$\begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta V_2 \end{bmatrix} = \begin{bmatrix} \partial P_2/\partial\delta_2 & \partial P_2/\partial\delta_3 & \partial P_2/\partial V_2 \\ \partial P_3/\partial\delta_2 & \partial P_3/\partial\delta_3 & \partial P_3/\partial V_2 \\ \partial Q_2/\partial\delta_2 & \partial Q_2/\partial\delta_3 & \partial Q_2/\partial V_2 \end{bmatrix}^{-1} \begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta Q_2 \end{bmatrix} \quad \text{——— (2)}$$

↑ Jacobian matrix elements.

$\frac{\partial P_2}{\partial \delta_2} \Rightarrow (p = q)$.

$$= \sum_{\substack{q=1 \\ \neq p}}^{n} |V_p||V_q||Y_{pq}| \cos(\theta_{pq} + \delta_q - \delta_p).$$

$= |V_2||Y_{21}||V_1| \sin(\theta_{21} + \delta_1 - \delta_2) + |V_2||Y_{23}||V_3| \sin(\theta_{23} + \delta_3 - \delta_2)$

$= [1.0 \times 2 \times 1.1 \times \sin 90^\circ] + [1.0 \times 5 \times 1.05 \times \sin 90^\circ]$

$= 2.2 + 5.25$

$= 7.45$

$$\frac{\partial P_2}{\partial \delta_3} \quad (p \neq q) = -|V_p||Y_{pq}||V_q| \sin(\theta_{pq} + \delta_q - \delta_p)$$

$= -|V_2||Y_{23}||V_3| \sin(\theta_{23} + \delta_3 - \delta_2)$

$= -1.0 \times 5 \times 1.05 \times \sin 90^\circ$

$= -5.25$

$$\frac{\partial P_3}{\partial \delta_2} \quad (p \neq q) = -|V_p||Y_{pq}||V_q| \sin(\theta_{pq} + \delta_q - \delta_p)$$

$= -1.05 \times 5 \times 1.0 \times \sin 90^\circ$

$= -5.25$

$$\frac{\partial P_3}{\partial \delta_3} \quad (p = q) = \sum_{\substack{q=1 \\ \neq p}}^{n} |Y_{pq}||V_q||V_p| \cos(\theta_{pq} + \delta_q - \delta_p)$$

$= |V_3||Y_{31}||V_1| \sin(\theta_{31} + \delta_1 - \delta_3) + |V_3||Y_{32}||V_2| \sin(\theta_{32} + \delta_2 - \delta_3)$

$= 1.05 \times 4 \times 1.1 \times \sin 90^\circ + 1.05 \times 5 \times 1.0 \times \sin 90^\circ.$

$= 4.62 + 5.25$

$= 9.87$

$$\frac{\partial P_2}{\partial V_2} = (p = q) = 2|V_p||Y_{pp}| \cos(\theta_{pp}) + \sum_{\substack{q=1 \\ \neq p}}^{n} |V_q||Y_{pq}| \cos(\theta_{pq} + \delta_q - \delta_p)$$

$= 2 \times 1 \times 6.9 \times \cos(-90^\circ) + 1.1 \times 2 \times \cos 90^\circ + 1.05 \times 5 \times \cos 90^\circ$

$= 0.$

$$\frac{\partial P_3}{\partial V_2} = p \neq q = |V_p||Y_{pq}|\cos(\theta_{pq}+\delta_q-\delta_p)$$

$$= |V_3||Y_{32}|\cos(\theta_{32}+\delta_2-\delta_3)$$

$$= 0.$$

$$\frac{\partial Q_2}{\partial \delta_2} = (p=q) = \sum_{\substack{q=1 \\ \neq p}}^{n} |V_p||V_q||Y_{pq}|\cos(\theta_{pq}+\delta_q-\delta_p)$$

$$= |V_2||Y_{21}||V_2|\cos(\theta_{21}+\delta_1-\delta_2) + |V_2||Y_{23}||V_3|\cos(\theta_{23}+\delta_3-\delta_2)$$

$$= 0.$$

$$\frac{\partial Q_2}{\partial \delta_3} = (p \neq q) = -|V_p||V_q||Y_{pq}|\cos(\theta_{pq}+\delta_q-\delta_p)$$

$$= -|V_2||Y_{23}||V_3|\cos(\theta_{23}+\delta_3-\delta_2)$$

$$= 0.$$

$$\frac{\partial Q_2}{\partial V_2} = (p=q) = -2|V_p||Y_{pp}|\sin\theta_{pp} - \sum_{\substack{q=1 \\ \neq p}}^{n} |V_q||Y_{pq}|\sin(\theta_{pq}+\delta_q-\delta_p)$$

$$= -2|V_2||Y_{22}|\sin\theta_{22} - |V_1||Y_{21}|\sin(\theta_{21}+\delta_1-\delta_2)$$

$$- |V_3||Y_{23}|\sin(\theta_{23}+\delta_3-\delta_2)$$

$$= -2 \times 1 \times 6.9 \times \sin(-90^\circ) - 1 \times 2 \times \sin 90^\circ - 1.05 \times 5 \sin 90^\circ.$$

$$= 13.8 - 2.0 - 5.25$$

$$= 6.55.$$

Jacobian Matrix Elements.

$$= \begin{bmatrix} 7.45 & -5.25 & 0 \\ -5.25 & 9.87 & 0 \\ 0 & 0 & 6.55 \end{bmatrix}$$

Eqⁿ. (2) becomes,

$$\begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta V_2 \end{bmatrix} = \begin{bmatrix} 7.45 & -5.25 & 0 \\ -5.25 & 9.87 & 0 \\ 0 & 0 & 6.55 \end{bmatrix}^{-1} \begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta Q_2 \end{bmatrix} \quad \text{——— (3)}$$

Calculate ΔP_2.

Δ means change (or) difference, so

Formula: $\boxed{\Delta P_p^{k+1} = P_{p,spec}^{k} - P_p^{k+1}}$

$P_{p,spec} \Rightarrow$ Value given in problem.

$P_p^{k+1} \Rightarrow$ Calculated Value.

$P_{p,spec}$:

$P_{p,spec} = P_G - P_L$

where P_G – Generator power.

P_L – Load power.

$P_{2,spec} = 0 - (-0.2)$

$= 0.2$

$-20 \Rightarrow -20/100$ in p.u.

$= -0.2$

$P_p^{k+1} = P_2' = 0.$

$\Delta P_2' = 0.2 - 0$

$\boxed{\Delta P_2' = 0.2 = \Delta P_2}$

Calculate ΔP_3:

$P_{3,spec} = 0.5 - 0$

$= 0.5$

$50 \Rightarrow 50/100$ in p.u

$= 0.5$ p.u.

$P_3' = 0$

$\therefore \Delta P_3' = 0.5 + 0$

$$\boxed{\Delta P_3' = 0.5}$$

Calculate ΔQ_2

$\Delta Q_2' = Q_{2,spec} - Q_2'$

$Q_{2,spec} = Q_G - Q_L$

$= 0 - 0.45$

$= -0.45$

$Q_2' = -0.55$

$\Delta Q_2' = -0.45 - (-0.55)$

$$\boxed{\Delta Q_2' = 0.1}$$

Subs ΔP_2, ΔP_3, ΔQ_2 in eq.(3)

$$\begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta V_2 \end{bmatrix} = \begin{bmatrix} 7.45 & -5.25 & 0 \\ -5.25 & 9.87 & 0 \\ 0 & 0 & 6.55 \end{bmatrix}^{-1} \begin{bmatrix} 0.2 \\ 0.5 \\ 0.1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.2147 & 0.1142 & 0 \\ 0.1142 & 0.162 & 0 \\ 0 & 0 & 0.1526 \end{bmatrix} \begin{bmatrix} 0.2 \\ 0.5 \\ 0.1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.1 \\ 0.1038 \\ 0.01526 \end{bmatrix}$$

$$\Delta\delta_2 = 0.1$$

$$\Delta\delta_3 = 0.1038$$

$$\Delta V_2 = 0.01526$$

$$\delta_2^1 = \delta_2^0 + \Delta\delta_2 = 0 + 0.1 = 0.1$$

$$\delta_3^1 = \delta_3^0 + \Delta\delta_3 = 0 + 0.1038 = 0.1038.$$

$$V_2 = V_2^0 + \Delta V_2 = 1 + 0.01526 = 1.01526.$$

Fast Decoupled Power Flow Method:

* Fast decoupled load flow method is faster, simple to program, more reliable, and requires less memory than NR load flow method.
* Fast decoupled method requires more iteration than NR method, but less time per iteration.

Procedure for Fast decoupled Power flow Method:

step :1 — step 12	: same from Newton Raphson method.

$V_3 = 1.04 - j0.004365$

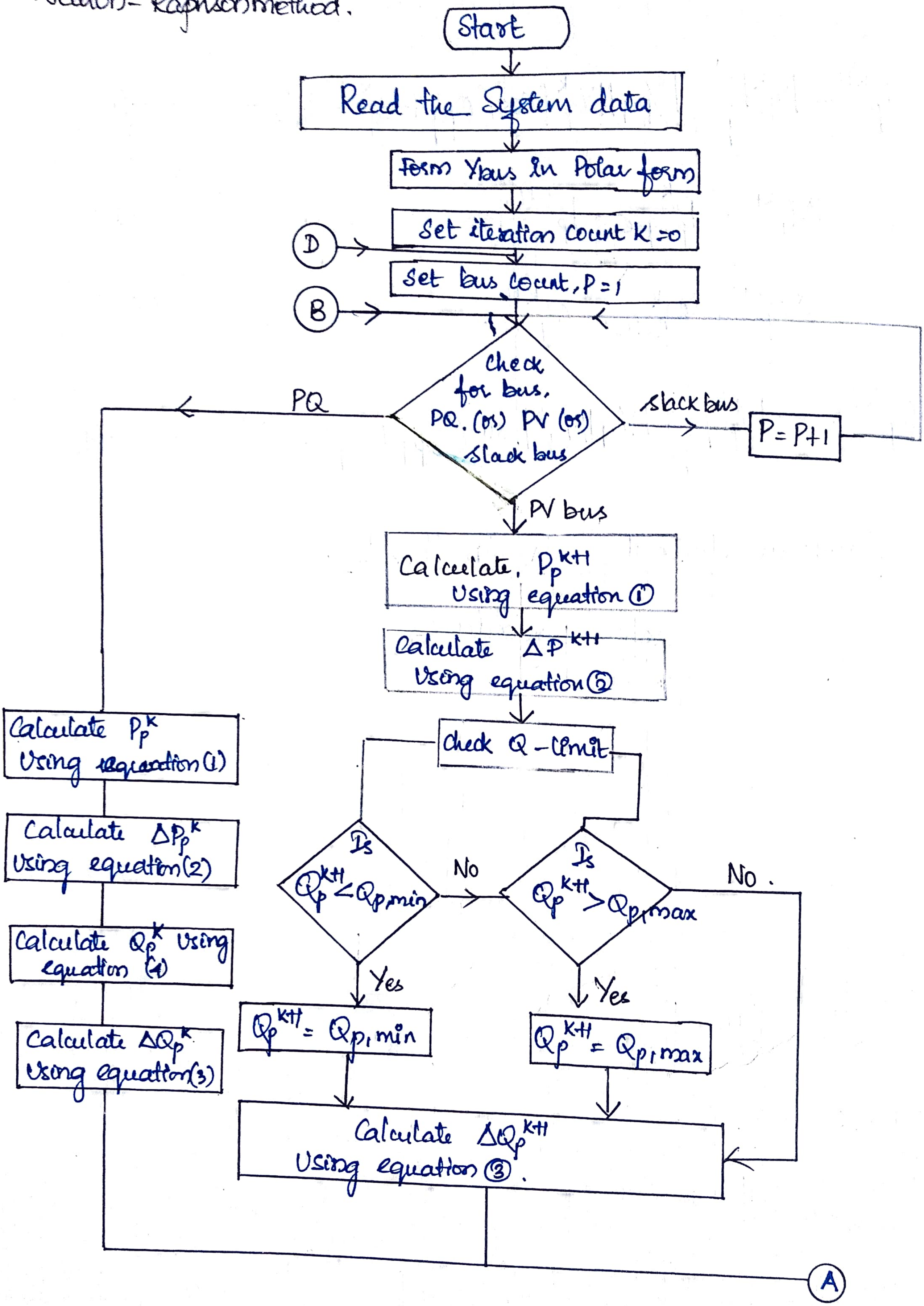
$V_4' = 1.0364 + 0.007426j$

Comparison between G-S, N-R & FDLF Methods.

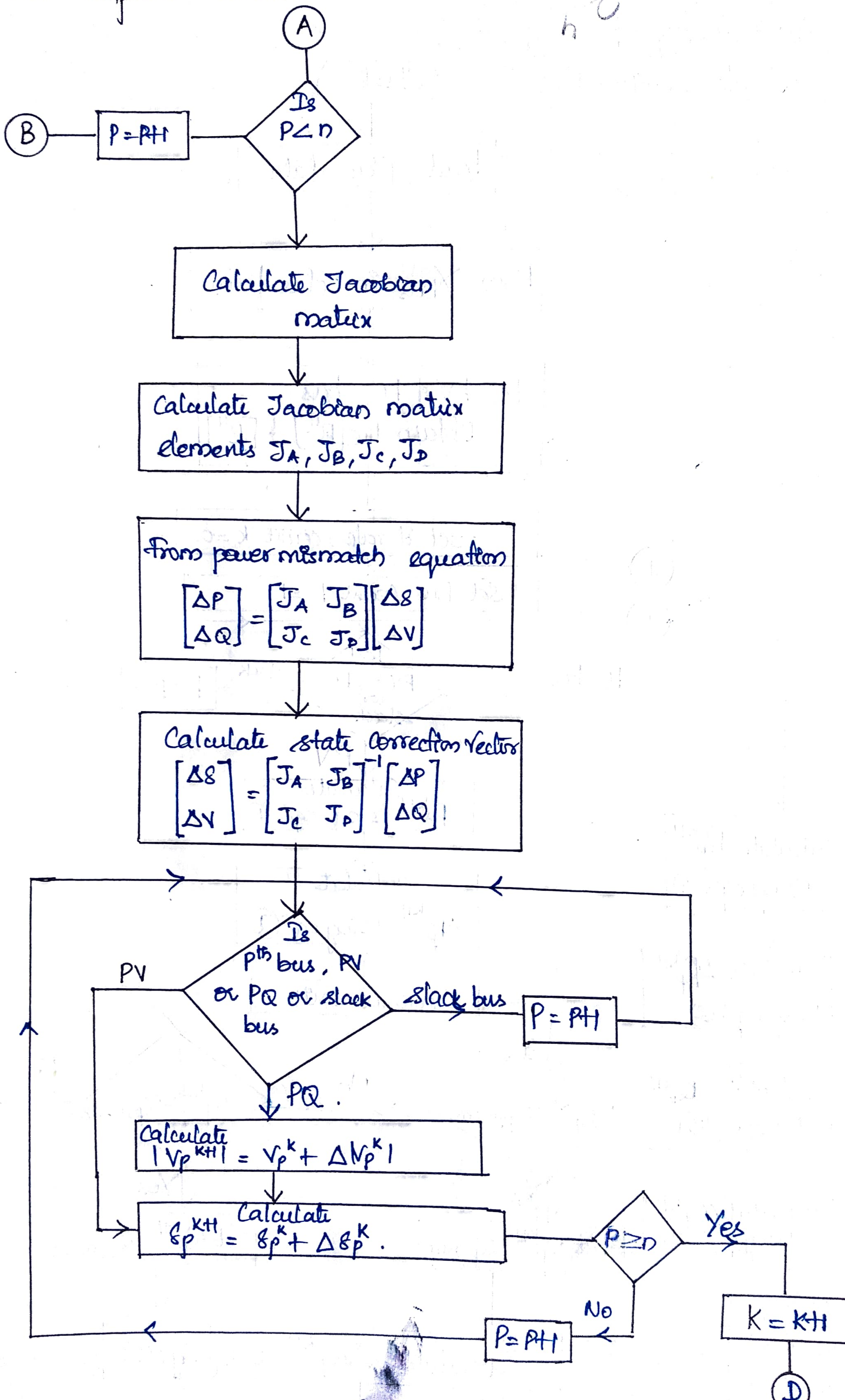
Gauss-Seidal	Newton Raphson	Fast decoupled
1. Rectangular Co-ordinates are preferred.	Polar Co-ordinates are preferred.	Polar Co-ordinates are prefered.
2. Time taken is less & arithmetic operation is less.	Time taken is 7 times than GS method.	5 times of N-R method 2/3 times of G-S method.
3. More iteration required.	3-5 iterations.	2-5 iterations.
4. More Computation cost.	Less Computation cost.	Less Computation cost.
5. Convergence uncertain.	Convergence certain	Convergence certain
6. Acceleration Factor required.	Not used	Not used.

Memory requirement is less.	Requires more memory.	Memory requirement is intermediate of G-S & N-R method.
Useful for small size systems.	For large size systems.	For large size systems.

... for
Newton-Raphson method.



Newton - Raphson method.



UNIT - III

FAULT ANALYSIS - BALANCED FAULTS

Fault:

A fault in a circuit is any failure which interferes with the normal flow of current.

Faults may be caused by following reasons.

* Insulation failure.
* Birds shorting the lines.
* Falling of a tree.
* Flashover of lines.
* Lightning.
* Accidental faulty operation, etc...

Types:

⇒ Balanced Faults (or) Symmetrical Faults.

⇒ Unbalanced Faults (or) Unsymmetrical Faults.

Balanced Faults (or) Symmetrical Faults:

A fault involving all the 3-ϕ is known as symmetrical or balanced fault.

The estimation of fault currents & voltages throughout the system during the fault is called fault calculations.

Importance of Fault Analysis:

Fault calculations are done for the following reasons.

* To select the ratings for fuses, breakers, relays.

* To check the MVA rating of the existing circuit breakers when new generators are added into the system.

* To design the grounding systems properly.

* For designing a substation short circuit current value should be known.

* For Economic planning and designing of power system.

* To calculate the reactance to limit fault current

* To calculate the magnitude of fault currents after fault occurs

Basic assumptions in Fault analysis of power systems:

Accurate solutions in fault analysis is practically impossible. Also the system impedances are also never known accurately. So some of the following assumptions are made.

* Load currents are considered negligible in comparison with fault currents.

* Shunt capacitances are transmission lines are neglected.

* No-load equivalent circuit of transformer are neglected.

* Resistance is neglected & only inductive reactance is taken into account.

* Each synchronous machine model is represented by an emf behind a reactance. ③

* The emfs of all generators are $1\angle 0$ per unit as assumption.

Balanced 3-ϕ Fault: (LLLG fault)

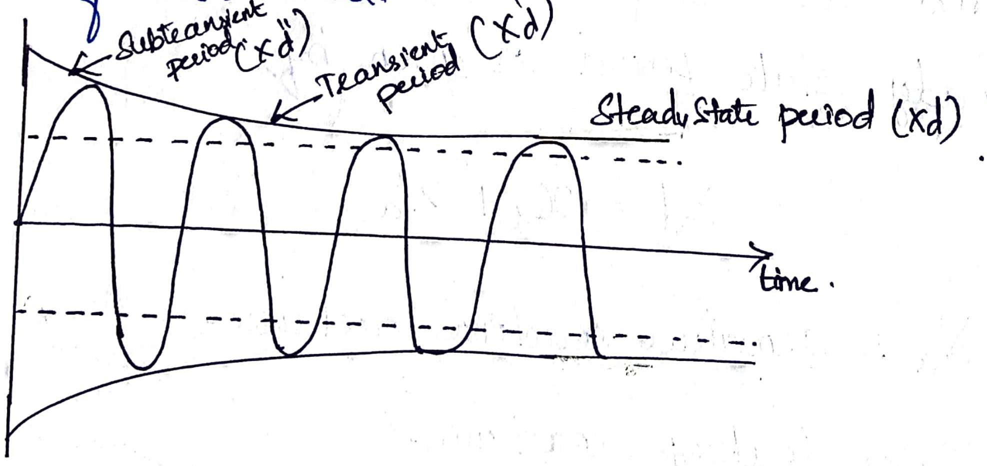
* Balanced 3-ϕ fault is defined as the simultaneous short circuit across all the 3 phases.

* Most severe type of fault, solved on per phase basis.

* The other 2 phases carry identical currents except for the phase shift.

Types of Reactances:

The symmetrical short circuit current can be divided into 3 regions (or) Period.



(i) The subtransient period.
(ii) The transient period.
iii) Steady state period.

Sub-transient Reactance:

The reactance produced by the machine in the initial period of short circuit fault, is called subtransient reactance of the machine.

$$X_d'' = X_l + \frac{1}{\left(\frac{1}{X_a} + \frac{1}{X_f} + \frac{1}{X_{dw}}\right)}$$

Transient Reactance:

After few cycles, the reactance by damper winding currents have died out, its called transient reactance.

$$X_d' = X_l + \frac{1}{\left(\frac{1}{x_a} + \frac{1}{x_f}\right)}$$

Steady State Reactance:

After transient period, the effect of field winding current will also die out, so the total reactance during steady state period is given by.

$$X_d = x_l + x_a .$$

X_a : armature reaction reactance.

X_l : leakage reactance.

X_f : Field reactance.

X_{dw} : damper winding reactance.

* The fault Analysis (or) Fault current calculation is done by any of the 3 following methods.

1. Network Reduction Technique (KCL & KVL).
2. Thevenin's theorem.
3. By Bus impedance Matrix.

* For Estimation of Subtransient fault current, the machine is represented by subtransient internal emf in series with subtransient reactance.

* For Estimation of transient fault current, the machine is represented by its transient internal emf in series with transient reactance.

* For Estimation of steady state current, the machine is represented by steady state internal emf in series with steady state reactance.

Symmetrical Fault Current Calculating Using Kirchoff's Voltage Law.

Procedure:

Step 1: choose base (kV_b, MVA_b) Values.

Step 2: Draw prefault condition reactance diagram.

Step 3: Calculate prefault Voltage at the fault point (V)

Step 4: Calculate prefault current (Load current) (I)

Step 5: Calculate internal emfs using (KVL) Kirchoff's Voltage Law.

Step 6: Draw the fault condition reactance diagram.

[This diagram is same as prefault reactance diagram, but the fault is represented by short circuit or fault impedance].

Step 7: Calculate the fault currents (in p.u) in the fault condition reactance diagram. (Using KVL).

Step 8: Calculate actual fault currents.

Actual Value = P.u Value × Base Value.

Note:

⇒ If there is no transformer, the Xp.u and base currents are same & vice-versa. for various sections of power system.

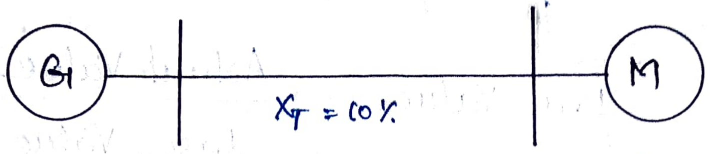
⇒ If power system is unloaded, then I = 0 & Voltage is 1pu

⇒ Internal emfs for subtransient, transient, steady state are same.

Problems Based on KVL.

1. A synchronous generator and motor are rated for 30,000 KVA 13.2 KV and both have subtransient reactance of 20%. The line connecting them has reactance of 10% on the base of machine ratings. The motor is drawing 20,000 kW at 0.8 leading. The terminal voltage of the motor is 12.8 KV. When a symmetrical 3-ϕ fault occurs at motor terminals, find subtransient current in generator, motor and at the

fault point.



$X_d'' = 20\%$

30MVA,

13.2KV

$X_d'' = 20\%$

20MW, 0.8 lead.

12.8KV

Step 1: Choose KV_b, MVA_b.

$KV_b = 13.2KV$

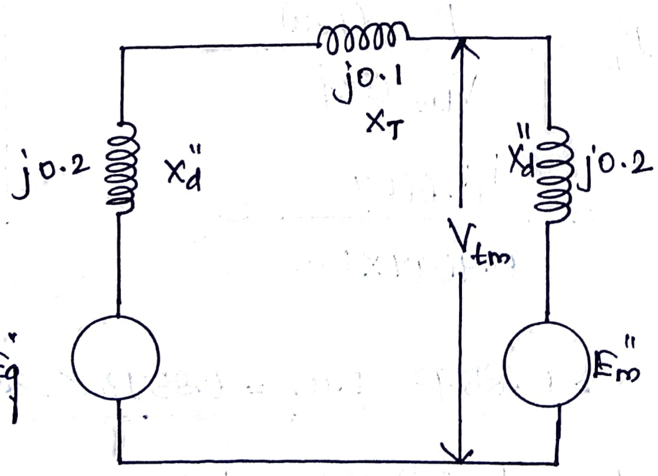
$MVA_b = 30MVA$.

Step 2: Prefault Reactance Diagram:

$X_{pu,new}$ of Generator $= 0.2j\ (p.u)$

$X_{pu,new}$ of ~~transformer~~ trn line $= 0.1j\ (p.u)$.

$X_{pu,new}$ of Motor $= 0.2j\ (p.u)$



Step 3: Calculate prefault Voltage at the fault point (Motor).

$V_{tm} = ?$

Actual value of prefault Voltage at fault point, $V_{tm} = 12.8$ kV.

$$\text{p.u value} = \frac{\text{Actual Value}}{\text{Base Value}}$$

$$= \frac{12.8}{13.2}$$

$$V_{tm} = 0.9697 \text{ p.u.}$$

Step : 4 : Calculate prefault current, (Load Current).

$$\text{Power in p.u} = \frac{\text{Actual Value}}{\text{Base Value}}$$

$$= \frac{20}{30}$$

$$= 0.6667 \text{ p.u.}$$

$$P = VI\cos\phi.$$

$$I_L = \frac{P \ (p.u)}{V_{tm} \cos\phi}$$

$$= \frac{0.6667}{0.9697 \times 0.8}$$

$\cos\phi = 0.$
$\phi = \cos^{-1}(0.$
$= 36.9$

0.6875

Prefault Current, $I_L = 0.8593$ p.u. $= 0.8593 \angle 36.9^\circ$ p.u.

Step 5: Calculate Internal emfs (E) using KVL:

Apply KVL in reactance diagram.

$$E_g'' = j0.2 I_L + j0.1 I_L + V_{tm}.$$

$E_g'' = j0.3 I_L + V_{tm}$

$= j0.3 \times (0.8593 \angle 36.9^\circ) + 0.9697$

$= j0.3 \times (0.6871 + 0.5159j) + 0.9697$

$= 0.8149 + j0.2061$

$E_g'' = 0.8405 \angle 14.19^\circ$. (p.u) ⇐ Internal emf of generator.

Internal emf of Motor, (KVL).

$V_{tm} = j0.2 I_L + E_m''$.

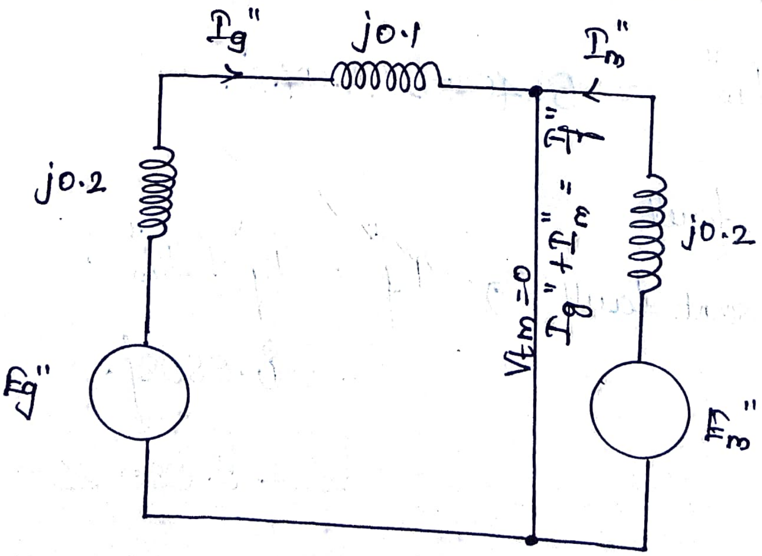
$E_m'' = V_{tm} - j0.2 I_L$.

$= 0.9697 - [j0.2 \times (0.8594 \angle 36.9^\circ)]$.

$= 1.0729 - j0.1375$

$E_m'' = 1.0817 \angle -7.3^\circ$ p.u.

Step 6: Draw fault condition reactance diagram.



Step 7: Calculate the fault currents in p.u.

Using KVL in loop 1,

$$E_g'' = j0.2 I_g'' + j0.1 I_g''$$

$$E_g'' = j0.3 I_g''$$

$$I_g'' = \frac{E_g''}{j0.3}$$

$$= \frac{0.8149 + j0.2062}{j0.3} = 0.6873 - 2.7163j$$

$$I_g'' = 2.801 \angle -75.8 \text{ (p.u)}$$

Using KVL in loop 2,

$$E_m'' = j0.2 I_m''$$

$$I_m'' = \frac{E_m''}{j0.2}$$

$$= \frac{1.0729 - j0.1375}{j0.2}$$

$$= -0.6873 - 5.3645j$$

$$I_m'' = 5.4085 \angle -97.3^\circ$$

The current in the fault during subtransient fault, $I_f'' = I_g'' + I_m''$

$$= -8.0808j$$

$$= 8.080 \angle -90^\circ \text{ p.u.}$$

Actual value = P.u Value × Base Value

(i) Actual value of subtransient fault current in generator } $I_g'' = 2.801 \angle -75.8 \times$ Base Value.

Base current value, $I_b = \frac{KVA_b}{\sqrt{3}\, KV_b}$.

$= \frac{30 \times 1000}{\sqrt{3} \times 13.2}$

$I_b = 1312.16\ A.$

Actual $I_g'' = 2.801 \angle -75.8 \times 1312.16$.

$= 3675.36 \angle -75.8$

$= 3.67 \angle -75.8\ KA.$

$||^{ly}$,

Actual $I_m'' = 5.4085 \angle -97.3° \times 1312.16$

$= 7096.81 \angle -97.3°$

$= 7.096 \angle -97.3°\ KA.$

$||^{ly}$,

Actual $I_f'' = 8.08 \angle -90° \times 1312.16$

$= 10602.25 \angle -90°$.

$= 10.6 \angle -90°\ KA.$

2. A alternator rated for 10MVA, 6.6 KV is supplying power to a motor of same rating through a transmission. The motor is drawing 8000 kW at 6.2KV at 0.9 power factor lagging before the occuring of fault. The subtransient reactance of both alternator & motor are 10% and the line has a subtransient reactance of 15% on the machine rating. Calculate the subtransient current supplied by generator & motor towards the fault point.

$MVA_b = 10MVA.$

$KV_b = 6.6KV$

$V_{tm} = 6.2kV.$

$V_{tm} = 0.9394$ p.u.

$I_L = 0.70967 \angle -25.842$ p.u.

$E_g'' = 1.0292 \angle 8.9254. = 1.01673 + j0.15967$

$E_m'' = 0.9107 \angle -4.0217 = 0.90846 - j0.063871.$

$I_g'' = 4.1168 \angle -81.075 = 0.6387 - j4.067.$

$I_m'' = 9.107 \angle -94.02 = -0.6387 - j9.08457.$

$I_f'' = 13.15 \angle -90°.$

Actual: $I_g'' = 3.6022 \angle -81.075$ KA.

$I_m'' = 7.9686 \angle -94.02$ KA.

$I_f'' = 11.506 \angle -90°$ KA.

Problems based on Thevenin's theorem.

Procedure for Symmetric Fault Calculation Using Thevenin's theorem:

Step 1: Choose base values KV_b, MVA_b.

Step 2: Determine the prefault condition reactance diagram.

Step 3: Calculate prefault Voltage at fault point. It is the thevenin's Voltage V_{th}.

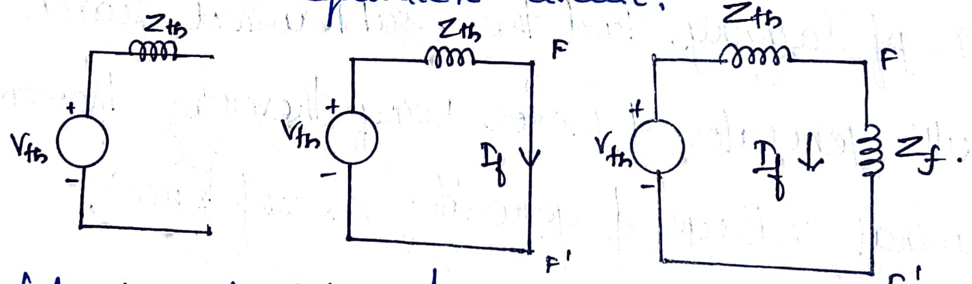
Step 4: Calculate the prefault current (ie. Load Current).

Step 5: Determine the thevenin's impedance at fault point.

[Z_{th}? * Replace all the sources by zero value source.

* Reduce the network to single equivalent impedance]

Step 6: Draw thevenin's equivalent circuit.



Step 7: Calculate per unit Value fault current I_f, is given by,

$$I_f = \frac{V_{th}}{Z_{th}+Z_f}$$

Step 8 The fault currents in other parts are calculated.

Fault current of Motor $= -I_L + \Delta I_m$ (− denotes direction).

Fault current of generator $= I_L + \Delta I_g$

Calculating change in current due to fault ($\Delta I_m + \Delta I_g$)

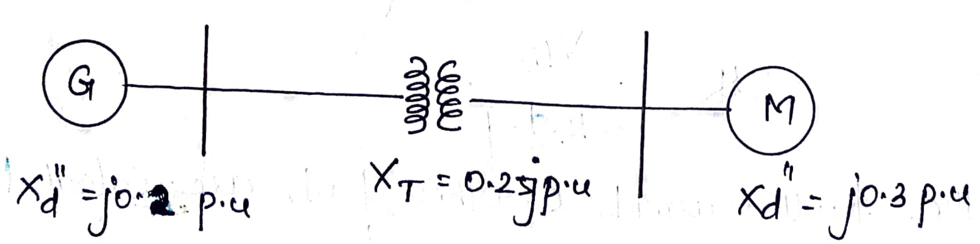
* Connect the thevenin's source with reversed polarity at fault.

* Replace all other sources to zero.

* Calculated currents in various parts of the sys are the change in current due to fault.

Step: 9: Convert to Actual Values. (Actual = p.u × Base).

3. A generator is connected through a transformer to a synchronous motor. The subtransient reactances of generator and motor are 0.2 p.u and 0.3 p.u respectively. The reactance of the transformer is 0.25 p.u. A 3-φ fault occurs at the terminals of the motor when the terminal voltage of the generator is 0.95 p.u. The output current of generator is 0.9 p.u with 0.95 pf lagging. Find the subtransient current in p.u at the fault generator & motor using thevenin's theorem. (Use the terminal voltage of generator as reference).



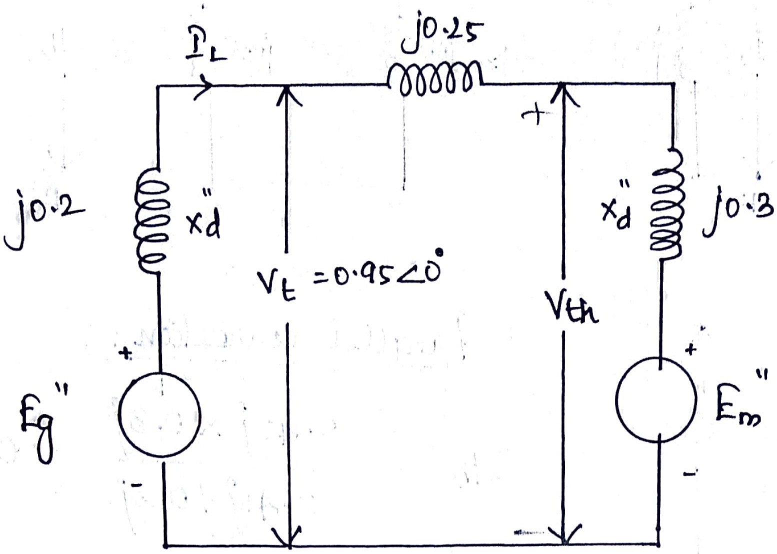
Given:

(V_{tg}) Terminal voltage of generator = 0.95 p.u.

output current of generator, I_L = 0.9 p.u.

cos (0.95)

Step :2 Prefault Condition reactance diagram.



Step : 3 : Calculate prefault voltage at fault point (motor). It is the thevenin's Voltage (V_{th}).

Apply KVL to the circuit to find V_{th}.

$$V_t = 0.25j I_L + V_{th}.$$

$$\therefore V_{th} = V_t - 0.25j I_L.$$

$$= 0.95\angle 0° - (0.25j \times 0.9\angle -18.1948).$$

$$= 0.95\angle 0° - (0.25j \times 0.855 - 0.2810j)$$

$$= 0.87975 - 0.21375j.$$

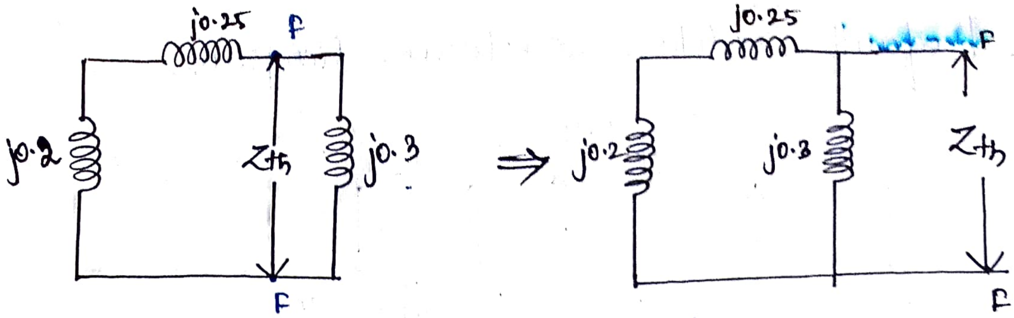
$$V_{th} = 0.9053\angle -13.656.$$

Step : 4 : $I_L = 0.9\angle -18.1948.$

Step 5 : Thevenin's Impedance (Z_{th})

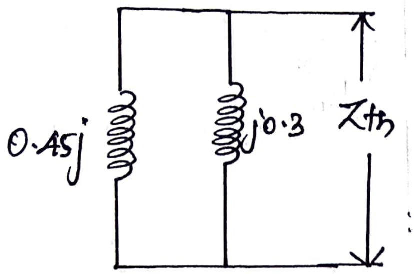
* all sources = 0.

* Reduce the network.



looking back from fault pt.

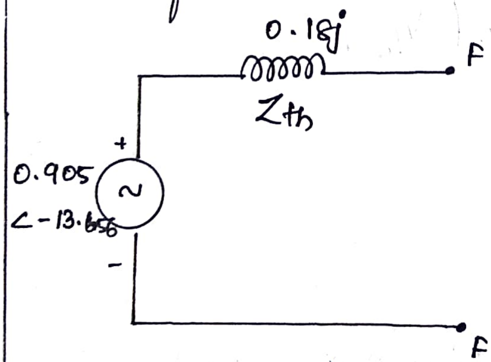
Parallel connection:



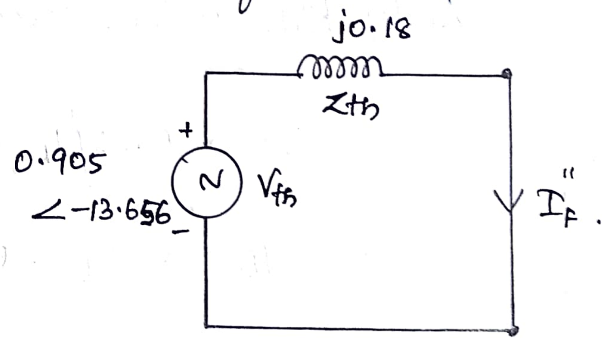
$$Z_{th} = \frac{0.45j \times 0.3j}{0.45j + 0.3j} = 0.18j \text{ (p.u.)}$$

step : 6 Draw thevenin's equivalent circuit,

Prefault Condition



Postfault condition



Step 7: Calculate fault current,

$$I_f'' = \frac{V_{th}}{Z_{th}} = \frac{0.905 \angle -13.656}{j0.18} = \frac{0.9053 \angle -13.656}{0.18 \angle 90^\circ}$$

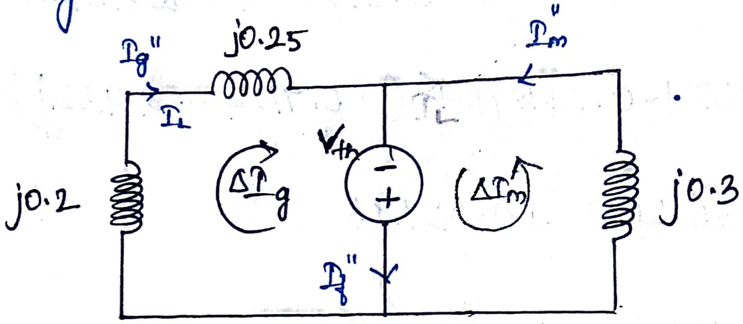
$$I_f'' = 5.029 \angle -103.656 \text{ p.u.}$$

step : 8 To calculate faults in motor & generator.

$$I_g'' = I_L + \Delta I_g.$$

$$I_m'' = -I_L + \Delta I_m.$$

$\Delta I_g = ?$ $\Delta I_m = ?$



* all other sources → 0
* thevenin's source in reverse polarity.

KVL for loop 1,

$$V_{th} = j0.2\,\Delta I_g + j0.25\,\Delta I_g$$

$$\Delta I_g = \frac{V_{th}}{0.2j + 0.25j}$$

$$= \frac{V_{th}}{0.45j}$$

$$= \frac{0.9053 \angle -13.656}{0.45j}$$

$$= \frac{0.9053 \angle -13.656}{0.45 \angle 90^\circ}$$

$$\Delta I_g = 2.0118 \angle -103.656.$$

KVL for Loop 2;

$$0.3j\,\Delta I_m = V_{th}$$

$$\Delta I_m = \frac{V_{th}}{0.3j}$$

$$= \frac{0.905339 \angle -13.656}{0.3 \angle 90^\circ}$$

$$\Delta I_m = 3.0177 \angle -103.656.$$

Subtransient fault current of motor is,

$$I_m'' = -I_L + \Delta I_m.$$

$= -(0.9\angle -18.1948) + 3.0177\angle -103.656.$

$= -(0.855 - 0.2810j) + (-0.7124 - 2.932j)$

$= -1.567478 - j2.65146$

$$I_m'' = 3.0801\angle -120.59\ (p.u).$$

subtransient fault current in generator,

$I_g'' = I_L + \Delta I_g.$

$= (0.9\angle -18.1948) + (2.0118\angle -103.656)$

$= 0.855 - 0.2810j + (-0.47496 - 1.9549j).$

$= 0.38004 - 2.2359j.$

$$I_g'' = 2.267\angle -80.353\ p.u.$$

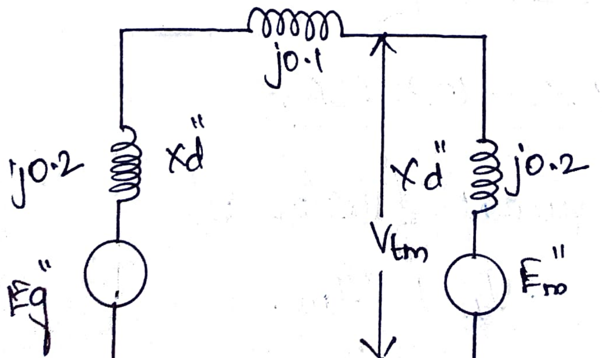
4. Problem No. 1 is solved using thevenin's Method:

Step :1 choose base values,

$KV_b = 13.2\ KV.$

$MVA_b = 30MVA.$

step :2 Prefault (before fault) Reactance diagram.



Step : 3 ~~calculate~~ calculate prefault voltage which is V_{th}, thevenin's Voltage.

$$V_{fm} = V_{th} = \frac{\text{Actual Value}}{\text{Base Value}}$$

$$= \frac{12.8}{13.2}$$

$$V_{th} = 0.9697 \text{ p.u.}$$

Step : 4 Calculate prefault current, Load current,

$$\text{Power in pu} = \frac{\text{Actual Value}}{\text{Base Value}}$$

$$= \frac{20}{30}$$

$$= 0.6667 \text{ p.u.}$$

$$P = VI\cos\phi.$$

$$I_L = \frac{P \text{ (p.u)}}{V_{fm}\cos\phi}$$

$$= \frac{0.6667}{0.9697 \times 0.8}$$

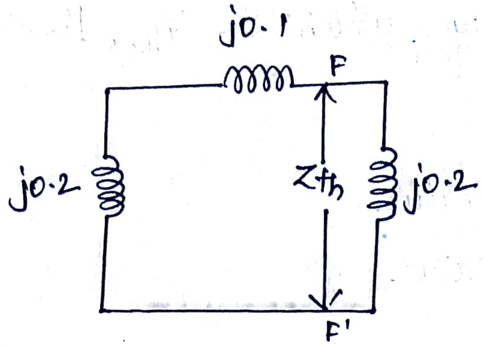
$$I_L = 0.8593 \text{ p.u}$$

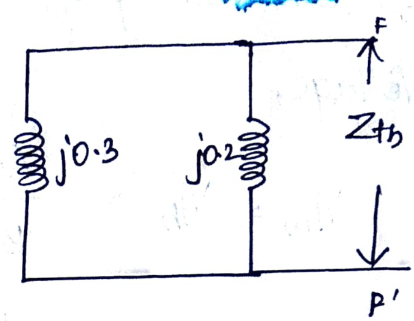
$$= 0.8593 \angle \cos^{-1}(0.8)$$

$$I_L = 0.8593 \angle 36.9° \text{ (p.u)}.$$

Step : 5 Calculate Z_{th}, thevenin's Impedance.

* Replace all sources to '0'.
* Reduce the n/w.



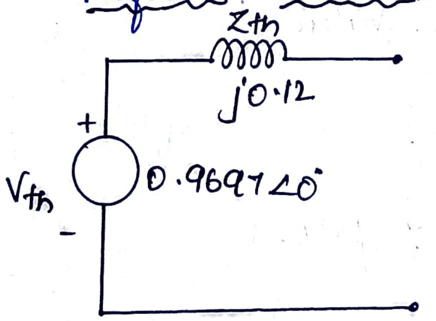


Parallel connection:

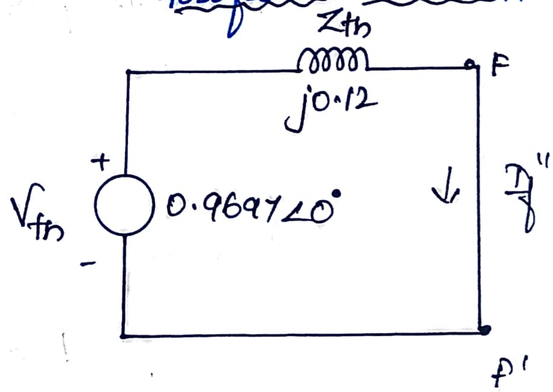
$$Z_{th} = \frac{j0.3 \times j0.2}{j0.3 + j0.2} = j0.12$$

Step : 6 : Draw thevenin's equivalent circuit,

Prefault Condition



Postfault condition.



Step : 7 calculate fault current Value,

$$I_f'' = \frac{V_{th}}{Z_{th}}$$

$$= \frac{0.9697\angle 0^\circ}{j0.12}$$

$$= \frac{0.9697\angle 0^\circ}{0.12\angle 90^\circ}$$

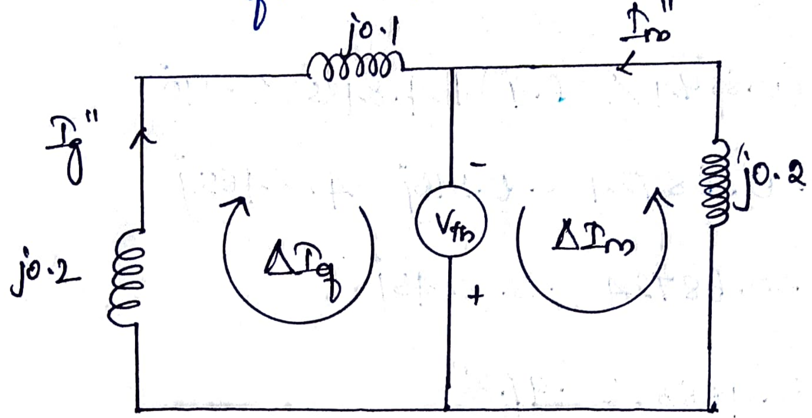
$$= 8.080\angle -90^\circ \text{ p.u.}$$

Step : 8 Faults currents in generator & Motor, I_g'', I_m''.

$$I_g'' = I_L + \Delta I_g \quad \text{——(1)}$$

$$I_m'' = -I_L + \Delta I_m \quad \text{——(2)}$$

Calculate, ΔI_g, ΔI_m,



* other source → 0.
* Polarity reversed.

$$V_{th} = j0.2\,\Delta I_g + j0.1\,\Delta I_g$$

$$\Delta I_g = \frac{V_{th}}{j0.2 + j0.1}$$

$$= \frac{0.9697\angle 0^\circ}{j0.3} = \frac{0.9697\angle 0^\circ}{0.3\angle 90^\circ}$$

$$\Delta I_g = 3.2323\angle -90^\circ$$

$$j0.2\,\Delta I_m = V_{th}$$

$$\Delta I_m = \frac{V_{th}}{j0.2}$$

$$= \frac{0.9695\angle 0^\circ}{0.2\angle 90^\circ}$$

$$\Delta I_m = 4.8485\angle -90^\circ$$

$$I_g'' = 0.8594\angle 36.9^\circ + 3.2323\angle -90^\circ$$

$$= 0.68724 + 0.516j + (-3.2323j).$$

$= 0.6872 - j2.7163$

$I_g'' = 2.802 \angle -75.8^\circ$ p.u

$I_m'' = -I_L + \Delta I_m$

$= -(0.8594 \angle 36.9^\circ) + 4.8485 \angle -90^\circ$

$= -0.68724 - 0.516j - 4.8485j$

$= -0.68724 - 5.3645j$

$I_m'' = 5.4083 \angle -97.3^\circ$.

Step : 9 Calculate Actual Value.

Base current, $I_b = \dfrac{KVA_b}{\sqrt{3} \times KV_b}$

$= \dfrac{30 \times 10^3}{\sqrt{3} \times 13.2}$

$I_b = 1312.16\,A$.

Actual $I_g'' = 2.802 \angle -75.8^\circ \times 1312.16$

$= 3.67667 \angle -75.8$ (KA)

Actual $I_m'' = 5.4083 \angle -97.3^\circ \times 1312.16$

$= 7.0992 \angle -97.3^\circ$ (KA).

Actual $I_f'' = 8.080 \angle -90^\circ \times 1312.16$

$= 10.60356 \angle -90^\circ$ (KA).

Selection of circuit Breakers:

* The circuit Breakers are protective devices which are used in power system to automatically open the faulty part of the system in the event of fault.

* The circuit Breakers are normally used in power system where power level is very high. So it has to interrupt heavy currents.

* Two ratings are required for the selection of circuit Breakers.

(i) Rated Momentary current.

(ii) Rated Symmetrical interrupting current.

Rated Momentary Current:

It is the maximum current that may flow through a circuit Breaker for a short duration.

It is calculated by multiplying the symmetrical subtransient fault current by a factor of 1.6.

The factor 1.6 accounts for dc-offset current during subtransient period.

Rated Symmetrical (or) short circuit interrupting current:

It is the maximum interrupting current at which the circuit Breaker will open its contacts due to fault.

Usually the circuit Breaker will open its contacts in the transient period (not in subtransient period).

* It is calculated by multiplying the transient short circuit current by a factor, depending on the speed of the breaker.

Speed of circuit Breaker	Multiplying Factor.
8 cycles or more	1.0
5 cycles	1.1
3 cycles	1.2
2 cycles	1.4
1½ cycles	1.5

* The speed of the circuit Breaker is the time between the occurrence of faults to the extinction of arc.

1 cycle for 50Hz frequency $= 1/50 = 0.02$ mse

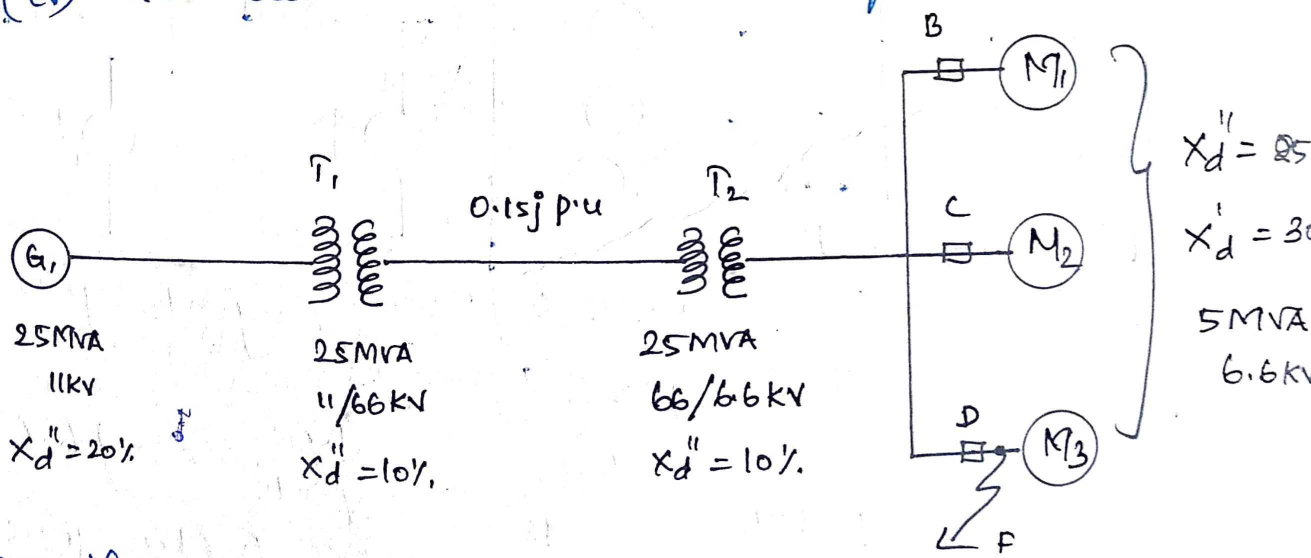
6\. (Univ. prob.) A 25 MVA, 11 KV generator with $X_d'' = 0.2$ per unit is connected through a transformer, line and a transformer to a bus that supplies three identical motors as shown. Each motor has $X_d'' = 0.25$ p.u and $X_d' = 0.3$ per unit on a base of 5 MVA, 6.6 KV. The 3ϕ rating of the step-up transformer is 25 MVA, 11/66 KV with a leakage reactance of 0.1 p.u and that of the step down transformer is 25 MVA, 33/6.6 KV with a leakage reactance of 0.1 p.u. The bus voltage at the motor is ... for the 66/6.6 KV, when a 3-ϕ fault occurs at point 'F'. For the specified fault calculate.

(i) The subtransient current in the fault.

(ii) The subtransient current in the breaker 'D'.

(iii) The momentary current in breaker "D" and.

(iv) The current to be interrupted by breaker "D" in 5 cy...



Base Values :

$MVA_b = 25$ MVA.

$KV_b = 11$ KV (for Generator)

$KV_b = 66KV$ (for line)

$KV_b = 6.6\,KV$ (for motor).

$X_{p.u,new}$ (or) $X_d'' = 0.2j$ for Generator.

$X_d'' = 0.1j$ for transformer. (T_1).

For transformer T_2,

$$X_d'' = 0.1 \times \left[\frac{66}{66}\right]^2 \times \left[\frac{25}{25}\right]$$

$$= 0.1j\ p.u.$$

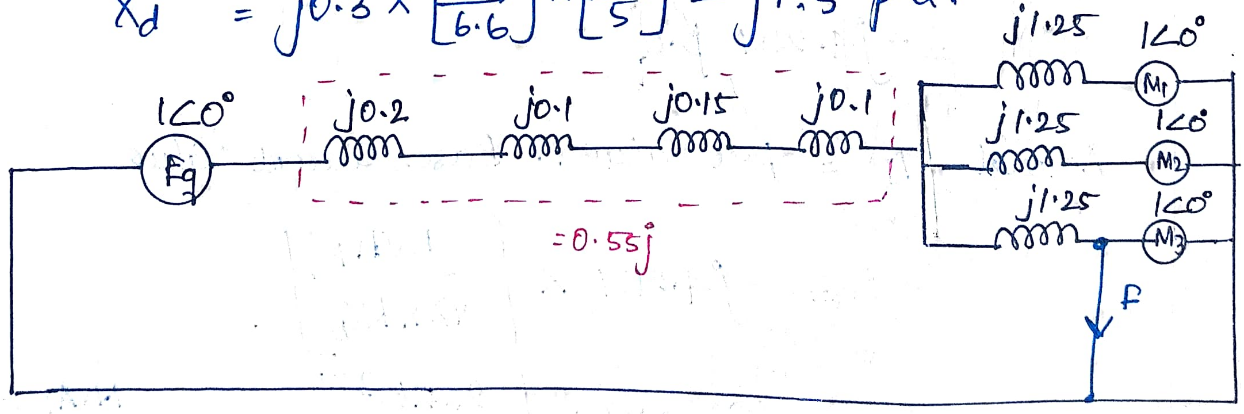
For Transmission line,

$$X_{p.u,new} = X_d'' = j0.15$$

For Motor (M_1, M_2, M_3)

$$X_d'' = j0.25 \times \left[\frac{6.6}{6.6}\right]^2 \times \left[\frac{25}{5}\right] = j1.25\ p.u.$$

$$X_d' = j0.3 \times \left[\frac{6.6}{6.6}\right]^2 \times \left[\frac{25}{5}\right] = j1.5\ p.u.$$



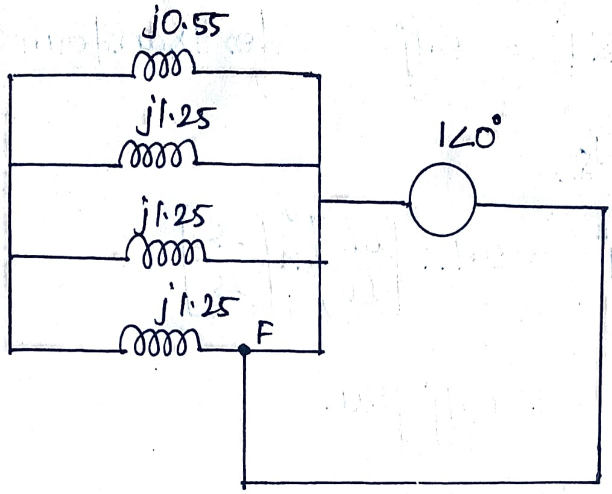
$V_{th} = 1\angle 0°$.

(emfs). $E_g = E_m = 1\angle 0°$

* No load, so there is no prefault current.

* Calculate Z_{th} at fault point, so the reactance diagram can be simplified to,

$j0.2$, $j0.1$, $j0.15$, $j0.1$ are in series. $= 0.55j$.



$Z_{th} = j0.2370$ p.u.

$$I_f = \frac{V_{th}}{Z_{th}}$$

$$= \frac{1\angle 0^\circ}{j0.2370}$$

$$= -j4.2181 \text{ p.u}$$

Actual I_f = p.u value × Base current value.

$$= -j4.2181 \times \left[\frac{KVA_b}{\sqrt{3} \times KV_b}\right]$$

$$= -j4.2181 \times \left[\frac{25 \times 10^3}{\sqrt{3} \times 6.6}\right]$$

$MVA_b = 25$

$\therefore KVA_b = 25 \times 10^3$.

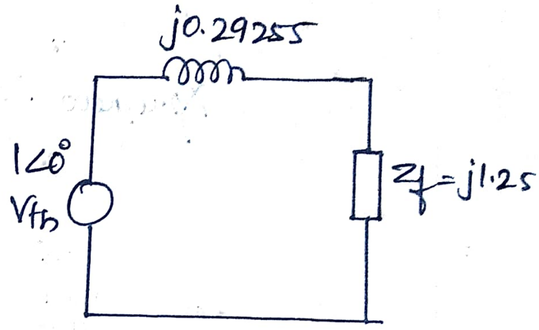
$$= -j4.2181 \times 2187$$

$$= -9224.7j = 9224\angle -90^\circ A.$$

(ii) Subtransient current in Breaker 'D'.

$Z_{th} = ?$ looking back impedance from fault point.

$$\frac{1}{Z_{th}} = \frac{1}{0.55j} + \frac{1}{1.25j} + \frac{1}{1.25j}$$



$$Z_{th} = j0.29255 \text{ p.u}$$

$$I_f = \frac{V_{th}}{Z_{th}}$$

$$= \frac{1\angle 0^\circ}{j0.29255}$$

$$I_f = -j3.41818 \text{ p.u}$$

Actual I_f = p.u Value × Base Value.

$$= -j3.41818 \times 2187$$

$$= -7475.56j$$

$$= 7475.56 \angle -90^\circ$$

$$= 7475.56 \text{ A}$$

(iii) Momentary current in Breaker 'D'.

= 1.6 × Symmetrical subtransient current at circuit Breaker 'D'.

$$= 1.6 \times 7475.56$$

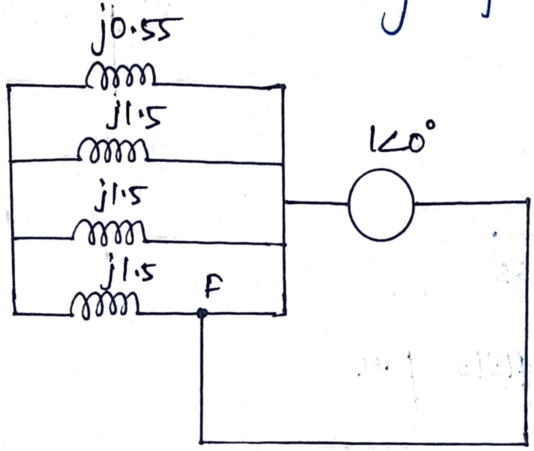
$$= 11.96 \text{ kA}.$$

(iv) Symmetrical short circuit interrupting current in 5 cycles: (by breaker 'D')

It occurs at transient period, so $X_d'' = j0.25$ is replaced by $X_d' = j0.3$ p.u.

$$X_{p.u,new} = X_d' = j0.3 \times \left[\frac{6}{6}\right]^2 \times \left[\frac{25}{5}\right]$$

$$= j1.5 \text{ p.u.}$$



$$\frac{1}{Z_{th}} = \frac{1}{j0.55} + \frac{1}{j1.5} + \frac{1}{j1.5}$$

$$Z_{th} = j0.31731.$$

$$I_f = \frac{V_{th}}{Z_{th}} = \frac{1\angle 0°}{j0.31731}$$

$$I_f = 3.1515 \text{ p.u}$$

Actual Current = 3.1515 × 2187

= 6892.3 A.

Interrupting current = 6892.3 × 1.1

= 7581.6 Amps.

[For 5 cycles Mly Factor = 1.1]

Short Circuit Capacity (SCC)

* The short circuit capacity of a bus of a network is defined as the product of the magnitudes of the prefault voltage and post fault current.
* It is also called fault level.

$$|SCC| = |V^0| \times |I_f| \quad (VA) \text{ ———— } (1)$$

V^0 = Prefault voltage in volts.

I_f = post fault currents in amps.

$$I_f = \frac{|V_{th}|}{|Z_{th}|} \text{ ———— } (2)$$

Let $V^0 = V_{th}$.

$$|SCC|_{uf} = |V_{th}||I_f|$$

$$= \frac{|V_{th}|^2}{|Z_{th}|} \quad [\text{subs. eq}^n. (2)] \text{ ———— } (3).$$

The Z_{th} is expressed in p.u as,

$$|Z_{th}|_{p.u} = \frac{|Z_{th}|}{Z_{base}} \text{ ———— } (4)$$

where $Z_{base} = \frac{V_{base}^2}{S_{base}}$ ——(5) $\left[= \frac{V_b \times V_b}{V_b \times I_b} = Z_b \right]$

S_{base} : base - voltampere in (VA) [power].

V_{base} : base voltage in amps.

Z_T : Thevenin's impedance in (ohms)

If V_{th} is chosen as V_b, then,

Subs (5) in eq^n (4) we get.

$$|Z_{th}|_{p.u} = |Z_{th}| \cdot \frac{S_b}{V_b^2}$$

$$|Z_{th}|_{p.u} = |Z_{th}| \frac{S_b}{|V_{th}^2|}$$

$$\frac{|V_{th}|^2}{|Z_{th}|} = \frac{S_b}{|Z_{th}|_{p.u}} \quad \text{——— (6).}$$

From eq^{n}s (3) & (6) we get,

$$|SCC|_{1\phi} = \frac{S_b}{|Z_{th}|_{p.u}} \quad \text{(VA/phase).}$$

Ily.

$$|SCC|_{3\phi} = \frac{S_{b,3\phi}}{|Z_{th}|_{p.u}}.$$

* A high short circuit capacity indicates that the system is stiff.
* A low short circuit capacity means that the system is weak.
* Both strength & severity of short circuit are met by a quantity known as short circuit capacity.
* A system having high short circuit capacity has a higher degree of voltage stability than the one with a low short circuit capacity.

7. Two generating stations having short circuit capacities of 1200 MVA and 800 MVA respectively and operating at 11 kV are linked by an interconnected cable having a reactance of 0.5 ohm/phase. Determine the short circuit capacity of each station.

Given:-

SCC of gen. 1 = 1200 MVA

SCC of gen. 2 = 800 MVA.

Choose, $MVA_b = 1200 = S_b$.

Station 1 :-

$$SCC = \frac{S_b}{|Z_{th}|}$$

$$1200 = \frac{1200}{|Z_{th}|}$$

$$|Z_{th}|_{p.u} = \frac{1200}{1200}$$

$$= 1 \text{ p.u.}$$

Station 2 :-

$$SCC = \frac{S_b}{|Z_{th}|}$$

$$800 = \frac{1200}{|Z_{th}|_{p.u}}.$$

$$|Z_{th}|_{p.u} = \frac{1200}{800}$$

$$= 1.5 \text{ p.u.}$$

Cable :-

$$\text{Base impedance} = \frac{(KV_b)^2}{MVA_b}.$$

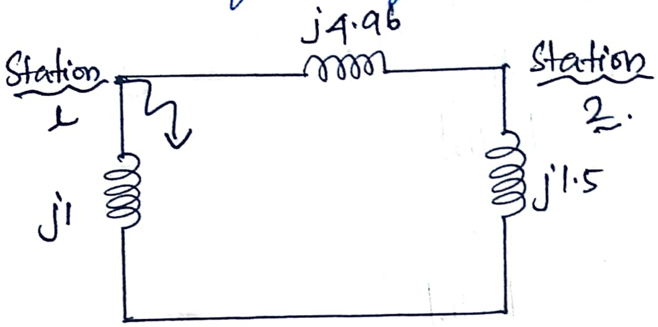
$$= \frac{11^2}{1200}$$

$= 0.1008$ (ohms).

$$\text{p.u impedance} = \frac{0.5}{0.1008}$$

$$= 4.96 \text{ p.u.}$$

Equivalent ckt is given by,

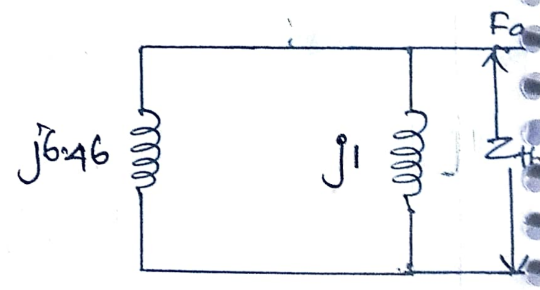


when fault occurs at Station 1;

Z_{th} ; looking back impedance from fault point. (j1)

j4.96 series j1.5 = j6.46.

j6.46 ‖ j1 = 0.8659j p.u.



$$\text{scc capacity of station} = \frac{S_b}{|Z_{th}|_{p.u}}$$

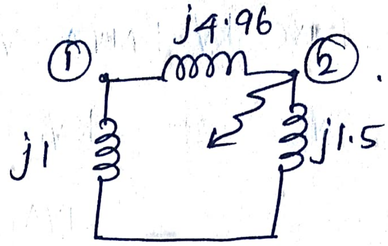
$$= \frac{1200}{0.8659}$$

$$= 1385.7 \text{ (MVA)}.$$

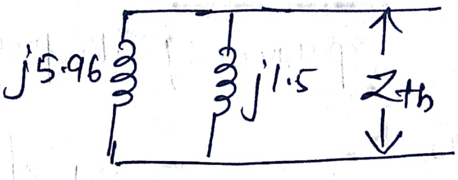
when fault occurs at station 2;

$j4.96$ series $j1$.

$j5.96$ $\|^{el}$ $j1.5$.



$$Z_{th} = \frac{j5.96 \times j1.5}{j5.96 + j1.5}$$



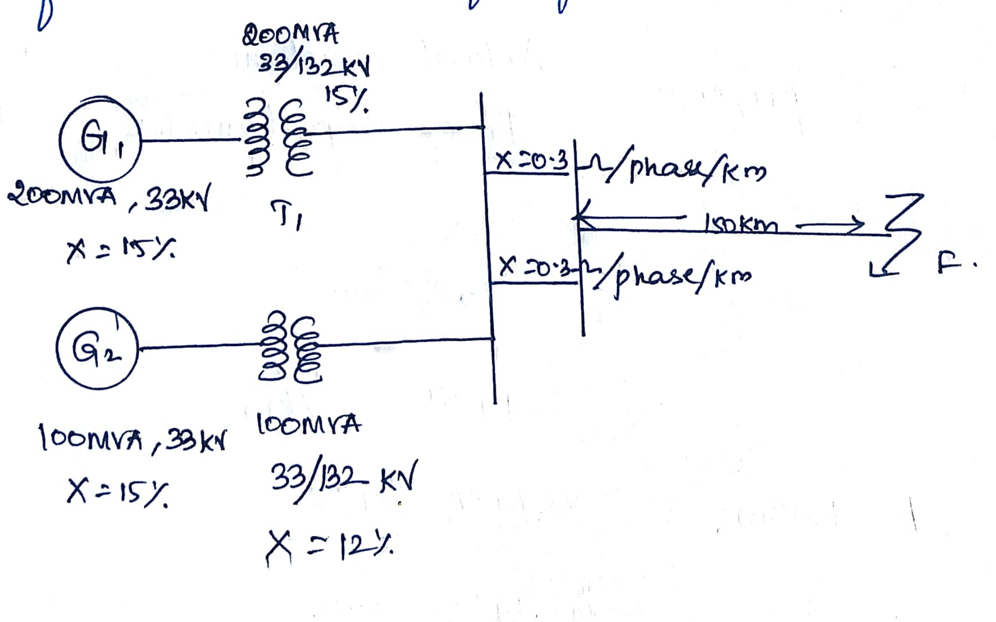
$$= j1.1984 \text{ p.u.}$$

$$SCC = \frac{S_b}{|Z|_{p.u}}$$

$$= \frac{1200}{1.1984}$$

$$= 1001.34 \text{ MVA.}$$

8. Figure shows 2 generating stations feeding a 132 KV system. Determine the total fault current, fault level and fault current supplied by each alternator for 3-ϕ solid fault at the receiving end bus. The length of the transmission line is 150km



Cheese Base KV_b & MVA_b Values,

$$KV_b = 33KV$$

$$MVA_b = 200 MVA.$$

$\underline{\underline{G_1}}$

$$X_{pu,new} = j0.15 \text{ p.u}$$

$\underline{\underline{G_2}}$ $$X_{pu,new} = j0.15 \times \left[\frac{33}{33}\right]^2 \times \left[\frac{200}{100}\right] = 0.3j \text{ p.u.}$$

$\underline{\underline{T_1}}$

$$X_{pu,new} = 0.15j \text{ p.u.}$$

$\underline{\underline{T_2}}$

$$X_{pu,new} = j0.12 \times \left[\frac{33}{33}\right] \times \left[\frac{200}{100}\right] = j0.24 \text{ p.u.}$$

Transmission line :-

HT side, $KV_{b,new}$ = KV_b on LT side $\times \frac{\text{HT voltage rating}}{\text{LT voltage rating}}$

$$= 33 \times \frac{132}{33}$$

$$KV_{b,new} = 132 KV$$

$$X_{pu,new} = \frac{\text{Actual reactance}(\Omega)}{\text{Base impedance}(\Omega)} = \frac{0.3}{\frac{(KV_{b,new})^2}{MVA_{b,new}}}$$

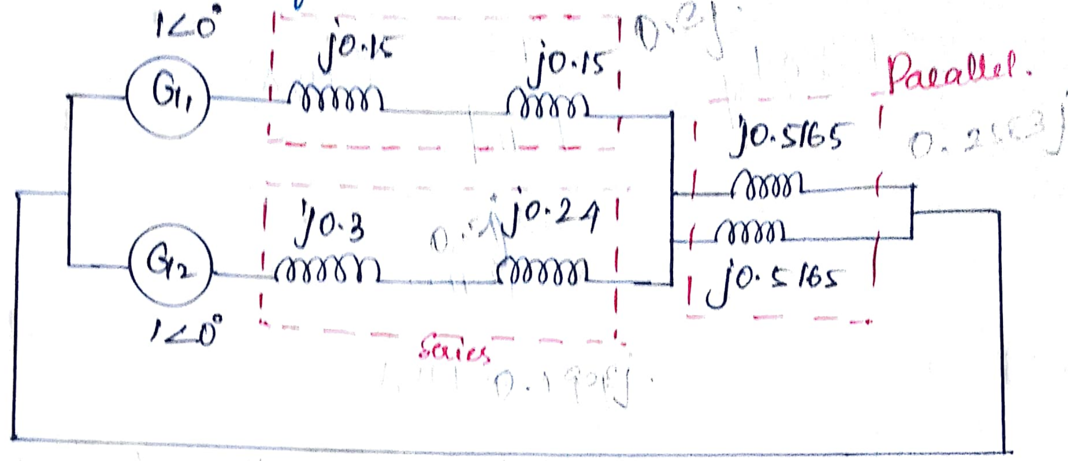
$$= \frac{0.3}{132^2/200}$$

$$= 3.44 \times 10^{-3} \cdot /km$$

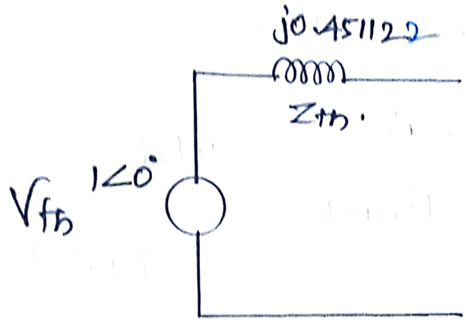
For 150Km, $= 3.44 \times 10^{-3} \times 150$

$$= j0.5165 \text{ p.u.}$$

Reactance diagram:



Z_{th}:



(i) Total Fault Current:

$$I_f = \frac{V_{th}}{Z_{th}} = \frac{1\angle 0^\circ}{j0.451122}$$

$$I_f\,(p.u) = -j2.2167\ p.u$$

Actual I_f = p.u × Base Current Value,

$$\text{Base Current } (I_b) = \frac{KVA_b}{\sqrt{3} \times KV_b} = \frac{200 \times 10^3}{\sqrt{3} \times 132} = 874.7731\ A.$$

$$\text{Actual } I_f = -j2.2167 \times 874.7731.$$

$$= -j1939.11\ \text{amps}.$$

$$= 1939.11 \angle -90^\circ.$$

$$|I_f| = 1.939\ KA.$$

(ii) Fault level (or) Short Circuit Capacity.

$$|SCC|_{3\phi} = \frac{S_b}{|Z_{th}|} \text{ (MVA)}.$$

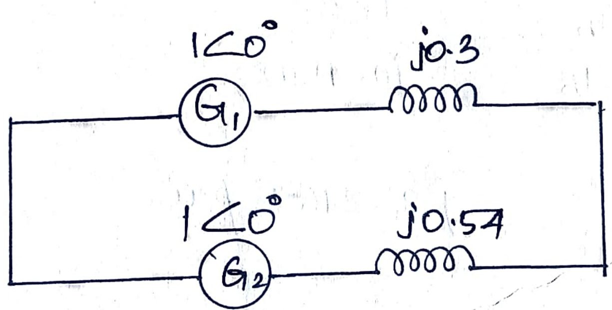
$$= \frac{200}{0.45122}$$

$$= 443.33 \text{ MVA}.$$

(iii) Fault Current supplied by each alternator: (I_{g_1}'', I_{g_2}'')

(using current division rule).

current flow through particular branch = $\frac{\text{Total Current} \times R \text{ in opp. branch}}{\text{Total Resistance}}$



Total fault current:

Actual = p.u × Base.

(For Generator) Base Current for 33KV side = $\frac{200 \times 1000}{\sqrt{3} \times 33}$ = 3499 Amps.

Actual = I_f'' pu × 3499 Amps.

$I_f = j2.2167$

Actual $I_f = j2.2167 \times 3499$

Total, Actual $I_f = 7756.432 \angle -90^\circ$ (Amps).

(I_{g1}'') fault current supplied by $= \dfrac{7756.43 \angle -90^\circ \times j0.54}{j0.3 + j0.54}$

$= 4986.278 \angle -90^\circ$ amps.

$|I_{g1}''| = 4.986$ kA.

(I_{g2}'') Fault current supplied by $G_2 = \dfrac{7756.43 \angle -90^\circ \times j0.3}{j0.3 + j0.54}$

$= 2770.1545 \angle -90^\circ$ A.

$|I_{g2}''| = 2.77$ kA.

Bolted Fault (or) Solid Fault:

If the fault impedance is zero, then the fault is called bolted fault.

DC offset Current:

The unidirectional transient component of short circuit current is called DC-offset current.

Fault Analysis Using Z-bus Matrix :

It is difficult to solve the large power systems using KVL & thevenin's method. So we go for fault analysis using Z-bus matrix.

Algorithm : (or) Procedure :

step : 1 Start.

step : 2 Read the line impedance, generator impedance and fault impedance, bus data, subtransient reactance.

step 3 : Form the Z_{bus} matrix using 4 types of modification (bus building alg.) (in

step 4 : Set all the bus voltages as $1\angle 0°$ p.u.

step 5 : Connect Z_f, fault impedance in series with faulted bus.

step 6 : Calculate fault current for i^{th} bus, using thevenin's equi. ckt.

$$I_f = \frac{V_q^0}{Z_{qq}+Z_f}.$$

step 7 : Calculate the generator current $I_G = \Sigma_{eq} I_{Gq}$.

step 8 : Calculate change in voltage ΔV_q and V_q^f.

Fault Voltages : $V_q^f = V_q^0 - Z_{qk} I_f$. K : fault bus no.

Find ΔV_q : Change in bus Voltages :

$$\begin{bmatrix} \Delta V_1 \\ \vdots \\ \Delta V_q \\ \vdots \\ \Delta V_N \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1q} & \cdots & Z_{1N} \\ \vdots \\ Z_{q1} & Z_{q2} & \cdots & Z_{qq} & \cdots & Z_{qN} \\ Z_{N1} & Z_{N2} & \cdots & Z_{Nq} & \cdots & Z_{NN} \end{bmatrix} \begin{bmatrix} 0 \\ \vdots \\ -I_q \\ \vdots \\ 0 \end{bmatrix}.$$

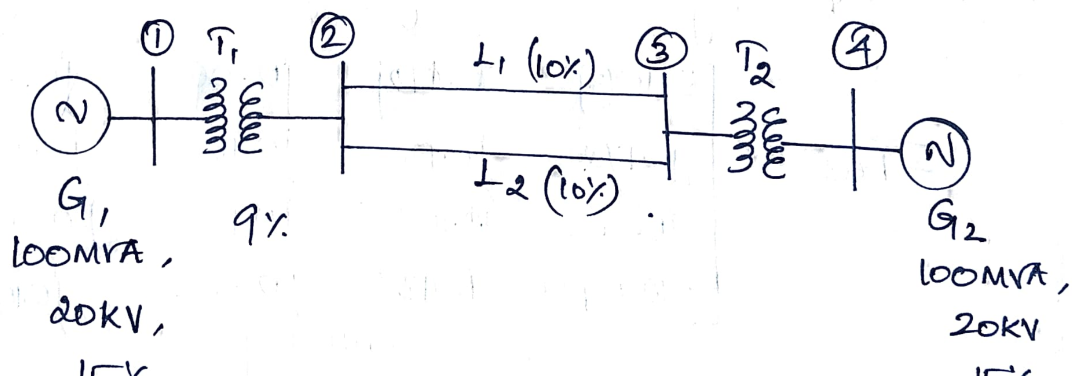
Step 9 : Calculate post-fault currents,

$$I^f_{pq} = \frac{V^f_p - V^f_q}{Z_{pq}}$$

Step 10 : Print the result.

Step 11 : Stop.

1. A symmetrical fault occurs on bus 4 of system shown in figure. Determine the fault current, post fault Voltages, & line currents.



Step : 1 To draw the reactance diagram & form Z_{bus}.

$KV_b = 20\,KV$.

$MVA_b = 100\,MVA$.

$X_{p.u,new}$ of $G_1 = 0.15j$ p.u.

$\underline{\underline{T_1}}$ $X_{pu,new} = 0.09j$ p.u.

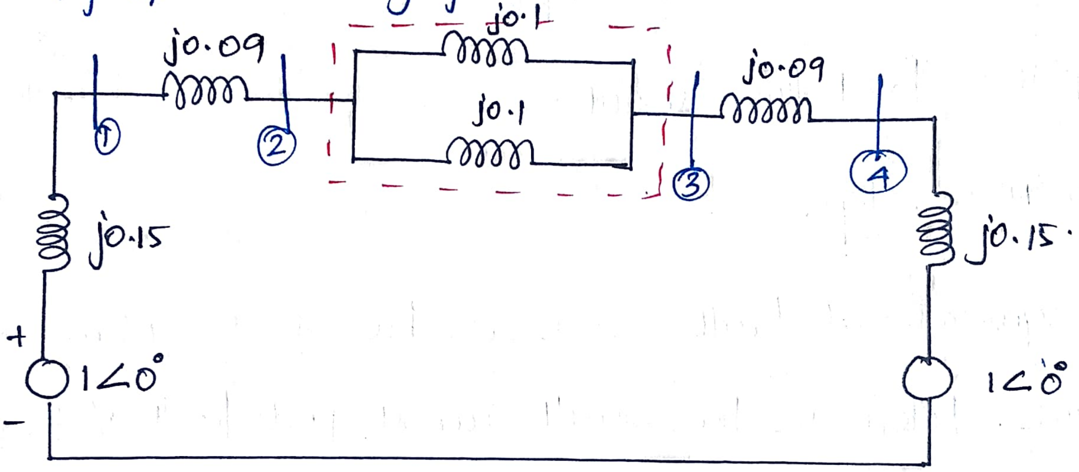
$\underline{\underline{L_1}}$ $X_{p.u,new} = 0.1j$ p.u.

$\underline{\underline{L_2}}$ $X_{pu,new} = 0.1j$ p.u

$\underline{\underline{T_2}}$

$X_{pu,new} = 0.09j$ p.u.

G_2 $X_{pu,new} = 0.15j$ p.u.



Form Zbus using bus building algorithm.

$$Z_{bus} = \begin{bmatrix} 0.1075j & 0.082j & j0.068 & j0.0424 \\ j0.082 & j0.13 & j0.108 & j0.068 \\ j0.068 & j0.108 & j0.13 & j0.082 \\ j0.0424 & j0.067 & j0.082 & j0.1075 \end{bmatrix}$$

$V_1^0 = 1\angle 0°$

$V_2^0 = 1\angle 0°$

$V_3^0 = 1\angle 0°$

$V_4^0 = 1\angle 0°$.

Calculate ~~fault~~ current, for fault (4) bus.

$$I_f = \frac{V_q^0}{Z_{qq}+Z_f} \quad (\text{or}) \quad \frac{V_{th}}{Z_{th}}$$

$$I_f = \frac{1\angle 0^\circ}{0.1075j}$$

$$= 9.3\angle -90^\circ \text{ p.u.}$$

$$\text{Actual } I_f = 9.3\angle -90^\circ \times \text{Base Current}.$$

$$= 9.3\angle -90^\circ \times \left[\frac{KVA_b}{\sqrt{3}\times KV_b}\right]$$

$$= 9.3\angle -90^\circ \times \left[\frac{100\times 10^3}{\sqrt{3}\times 20}\right]$$

$$= 26.85\angle -90^\circ \text{ kA}.$$

Post fault Voltages:-

$$V_1^f = V_1^0 - Z_{14} I_f.$$

$$= 1\angle 0^\circ - j0.042\times 9.3\times \angle -90^\circ$$

$$= 0.6056 \text{ p.u.}$$

$$V_2^f = V_2^0 - Z_{24} I_f$$

$$= 1\angle 0^\circ - j0.068\times 9.3\angle -90^\circ$$

$$= 0.3686 \text{ p.u.}$$

$$V_3^f = V_3^0 - Z_{34} I_f$$

$$= 1\angle 0^\circ - j0.082\times 9.3\angle -90^\circ.$$

$$= 0.2374 \text{ p.u.}$$

$V_4^f = 1\angle 0 - j0.1075 \times 9.3 \angle -90°$

$= 0$ p.u.

12, 23, 34.

Post fault currents,

$$I_{pq} = \frac{V_p^f - V_q^f}{Z_{pq}}$$

I_{13}, I_{23}, I_{34} :

$$I_{13} = \frac{V_1^f - V_3^f}{Z_{13}} = \frac{0.6056 - 0.3886}{j0.09} = 2.634 \text{ p.u.}$$

$$I_{23} = \frac{V_2^f - V_3^f}{Z_{23}} =$$

$$I_{34} =$$

1. The bus impedance matrix of 4 bus s/m with values in p.u is given by

$$Z_{bus} = \begin{bmatrix} 0.1488j & 0.0651j & 0.864j & 0.0978j \\ 0.0651j & 0.1554j & 0.0799j & 0.0967j \\ 0.0864j & 0.0798j & 0.1341j & 0.1058j \\ 0.0978j & 0.0967j & 0.1058j & 0.1566j \end{bmatrix}.$$

If a 3-ϕ fault occurs at bus-1 when there is no-load, find the subtransient current in the fault and voltages at all buses. Also find the subtransient current supplied by the generator connected to bus-2 by taking the subtransient reactance of generator as 0.2j p.u. The prefault voltage is 1 p.u.

i) Subtransient current during fault:

$$I_f'' = \frac{V_q^0}{Z_{qq}+Z_f}$$

$V_1^0 = V_2^0 = V_3^0 = V_4^0 = 1\angle 0^\circ$.

Fault occurs at '1'

$$I_f'' = \frac{1\angle 0^\circ}{Z_{11}}$$

$$= \frac{1\angle 0^\circ}{j0.1488}$$

$$= -6.7204j$$

$$= 6.72\angle -90^\circ \text{ p.u.}$$

ii) Voltages at all buses:- $(V_1^f, V_2^f, V_3^f, V_4^f)$.

$$V_q^f = V_q^0 - Z_{qK} I_f$$

K = 1 : fault bus no.

$$V_1^f = V_1^0 - Z_{11} I_f''$$

$$= 1 - (0.1488j) \times (-6.7204j)$$

$$= 0.$$

$$V_2^f = V_2^0 - Z_{21} I_f''$$

$$= 1 - (0.0657j)(-6.7204j)$$

$= 0.5625$ p.u

$V_3^f = V_3^0 - Z_{31} I_f$

$= 1 - 0.0864j \times 6.7204\angle -90^\circ$

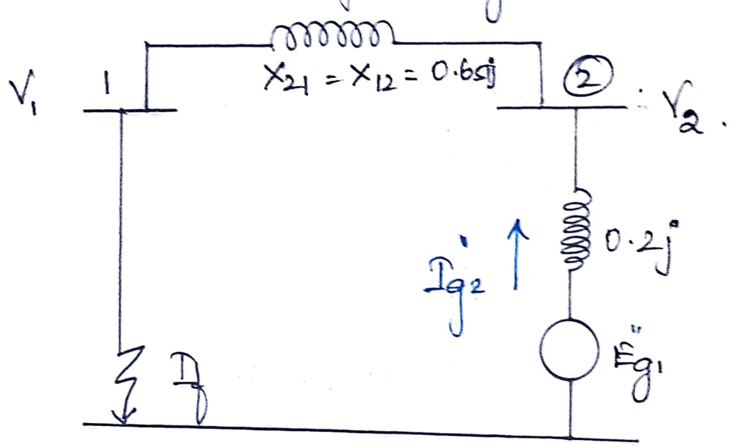
$= 0.4194$ p.u.

$V_4^f = V_4^0 - Z_{41} I_f$

$= 1 - 6.7204\angle -90^\circ \times 0.0978j$.

$= 0.3427$ p.u.

Subtransient Current Supplied by Generator:



$E_{g1}'' = I_{g2}'' 0.2j + V_2$

$I_{g2}'' = \frac{E_{g2}'' - V_2}{0.2j}$

$E_{g2}'' = 1$ p.u
[no current, so no potential drop]

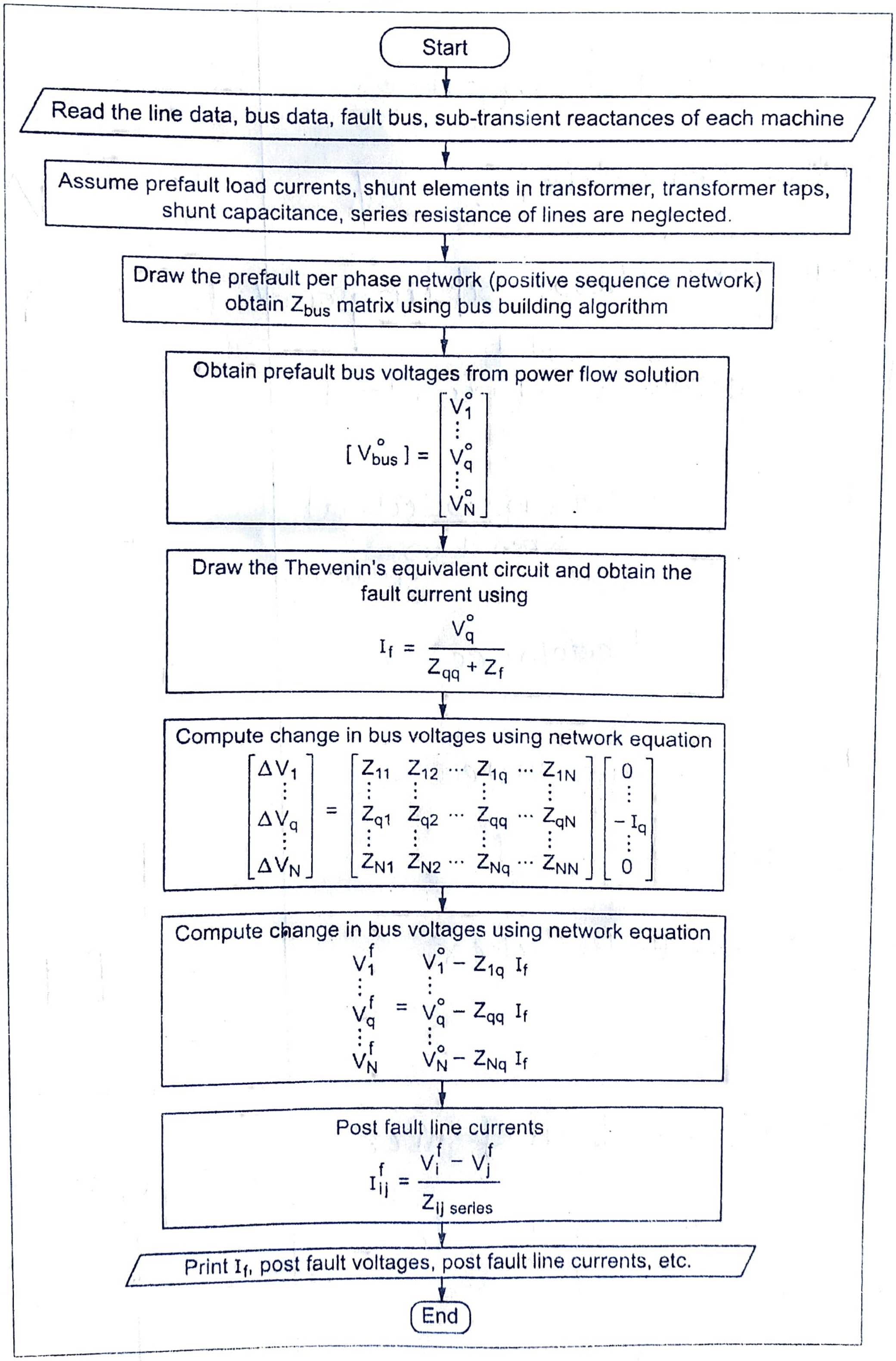
$= \frac{1 - 0.5625}{0.2j}$

$= -2.1875j$.

$= 2.1875\angle -90^\circ$.

very simple and practical. Thus all fault calculations are formulated in the bus frame of reference using bus impedance matrix Z_{bus}.

Symmetrical Fault Analysis using Z_{bus} (Flow chart)



Unit-IV

Fault Analysis Unbalanced Faults

Symmetrical Components:

The unbalanced (or) unsymmetrical system of n related vectors can be resolved into 'n' system of balanced vectors called symmetrical components.

In a 3-ϕ system, the 3-unbalanced vectors ie V_a, V_b, V_c or I_a, I_b, I_c can be resolved into 3 balanced system of vectors. The vectors of the balanced system are called symmetrical components. They are,

1) Positive sequence components.
2) Negative sequence components.
3) Zero sequence components.

Positive Sequence Components:

The positive sequence components consists of 3 vectors equal in magnitude, displaced from each other by 120° in phase, and having the same phase sequence as the original vectors (abc).

V_{a1}, V_{b1} & V_{c1} — Positive sequence components of V_a, V_b & V_c with phase sequence abc.

I_{a1}, I_{b1}, & I_{c1} — Positive sequence components of I_a, I_b, I with phase sequence abc.

Negative Sequence Components:

It consists of 3 Vectors equal in magnitude, displaced from each other by 120° in phase and having the phase sequence opposite to that of the original vectors. (acb)

V_{a2}, V_{b2}, V_{c2} – Negative sequence components of V_a, V_b, with phase sequence acb.

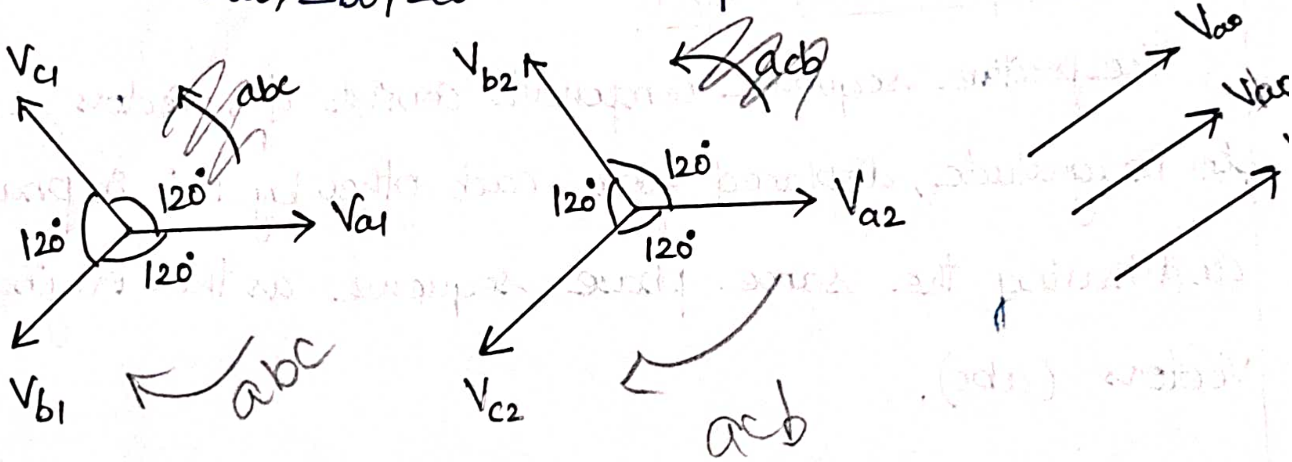
I_{a2}, I_{b2}, I_{c2} – Negative sequence currents of I_a, I_b, I with phase sequence acb.

Zero sequence Components:

It consists of 3 Vectors equal in magnitude, with zero displacement, from each other.

V_{a0}, V_{b0}, V_{c0} – Zero sequence components of V_a, V_b, V_c.

I_{a0}, I_{b0}, I_{c0} – Zero sequence currents of I_a, I_b, I



* On rotating V_{a1} by 120° anticlockwise we get V_{c1}: [$V_{c1} = a$
* On rotating V_{a1} by 240° anticlockwise, we get V_{b1} [$V_{b1} = a^2$
* On rotating V_{a2} by 120° anticlockwise, we get V_{b2} [$V_{b2} =$
* On rotating V_{a2} by 240° anticlockwise, we get V_{c2} [$V_{c2} =$

WKT, $a = 1\angle 120^\circ$; $a^2 = 1\angle 240^\circ$; $a^3 = 1\angle 360^\circ = 1$

i.e. $1 + a + a^2 = 0$.

Unbalanced Vectors From Symmetrical Components

* Each of the original unbalanced vector is the sum of its positive, negative and zero sequence component.

$$\left.\begin{aligned} V_a &= V_{a0} + V_{a1} + V_{a2} \\ V_b &= V_{b0} + V_{b1} + V_{b2} \\ V_c &= V_{c0} + V_{c1} + V_{c2} \end{aligned}\right\} \quad \text{—— (1)}$$

WKT,

$V_{b0} = V_{a0}$ $\quad V_{b1} = a^2 V_{a1}$ $\quad V_{b2} = a V_{a2}$

$V_{c0} = V_{a0}$ $\quad V_{c1} = a V_{a1}$ $\quad V_{c2} = a^2 V_{a2}$

subs. above in eqn 1, 2, 3, we get,

$$\left.\begin{aligned} V_a &= V_{a0} + V_{a1} + V_{a2}. \\ V_b &= V_{a0} + a^2 V_{a1} + a V_{a2} \\ V_c &= V_{a0} + a V_{a1} + a^2 V_{a2} \end{aligned}\right\} \quad \text{—— (2)}$$

The above equations can be arranged in matrix form,

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} \quad \text{—— (3)}$$

* From the above eqn. unbalanced voltage vectors can be calculated from symmetrical components.

Symmetrical Components From Unbalanced Vectors

In short eqn. (3) can be written as,

$$V = A V_{sy} \quad \text{—————— (4)}$$

where, $V = \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$; unbalanced Voltage vectors.

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$$

$V_{sy} = \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}$; Symmetrical Components.

Premultiply Eqn. by A^{-1}.

$$A^{-1} V = V_{sy}$$

$$V_{sy} = A^{-1} V$$

Calculate A^{-1}.

$$A^{-1} = \frac{\text{Adjoint of } A}{\text{Determinant of } A}$$

Determinant of A $= \begin{vmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{vmatrix}$

$$= 1(a^4 - a^2) - 1(a^2 - a) + 1(a - a^2)$$

$$a^4 = a^3 . a = a \quad [a^3 = 1].$$

$= (a-a^2) + (a-a^2) + (a-a^2)$

$\Delta_A = 3(a-a^2).$

Adjoint of A.

$$= \begin{bmatrix} \Delta_{11} & \Delta_{12} & \Delta_{13} \\ \Delta_{21} & \Delta_{22} & \Delta_{23} \\ \Delta_{31} & \Delta_{32} & \Delta_{33} \end{bmatrix}$$

$\Delta_{11} = \begin{vmatrix} a^2 & a \\ a & a^2 \end{vmatrix} = a^4 - a^2 = a^3 . a - a^2 = a - a^2.$

$\Delta_{12} = -\begin{vmatrix} 1 & a \\ 1 & a^2 \end{vmatrix} = -(a^2 - a) = a - a^2.$

$\Delta_{13} = \begin{vmatrix} 1 & a^2 \\ 1 & a \end{vmatrix} = a - a^2 =$

$\Delta_{21} = -\begin{vmatrix} 1 & 1 \\ a & a^2 \end{vmatrix} = -(a^2 - a) = a - a^2.$

$\Delta_{22} = \begin{vmatrix} 1 & 1 \\ 1 & a^2 \end{vmatrix} = a^2 - 1$

$\Delta_{23} = -\begin{vmatrix} 1 & 1 \\ 1 & a \end{vmatrix} = -(a-1) = 1 - a.$

$\Delta_{31} = \begin{vmatrix} 1 & 1 \\ a^2 & a \end{vmatrix} = a - a^2$

$\Delta_{32} = -\begin{vmatrix} 1 & 1 \\ 1 & a \end{vmatrix} = -(a-1) = 1 - a.$

$\Delta_{33} = \begin{vmatrix} 1 & 1 \\ 1 & a^2 \end{vmatrix} = a^2 - 1$

$$\text{Adjoint of } A = \begin{bmatrix} a-a^2 & a-a^2 & a-a^2 \\ a-a^2 & a^2-1 & 1-a \\ a-a^2 & 1-a & a^2-1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{\Delta}\begin{bmatrix} a-a^2 & a-a^2 & a-a^2 \\ a-a^2 & a^2-1 & 1-a \\ a-a^2 & 1-a & a^2-1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{3(a-a^2)}\begin{bmatrix} a-a^2 & a-a^2 & a-a^2 \\ a-a^2 & a^2-1 & 1-a \\ a-a^2 & 1-a & a^2-1 \end{bmatrix}$$

Taking $a-a^2$ common outside,

$$= \frac{\cancel{a-a^2}}{3(\cancel{a-a^2})}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2-1/a-a^2 & 1-a/a-a^2 \\ 1 & 1-a/a-a^2 & a^2-1/a-a^2 \end{bmatrix} \text{———} (5)$$

$$\frac{a^2-1}{a-a^2} = \frac{a^2-a^3}{a-a^2} \quad [a^3=1].$$

$$= \frac{a(\cancel{a-a^2})}{\cancel{a-a^2}}$$

$$\frac{a^2-1}{a-a^2} = a. \text{———} (A)$$

$$\frac{1-a}{a-a^2} = \frac{a^2(1-a)}{a^2(a-a^2)} \quad \text{Xly by } a^2 \text{ both num. \& den.}$$

$$= \frac{a^2(1-a)}{a^3-a^4}$$

$$= \frac{a^2(\cancel{1-a})}{a(\cancel{1-a})}$$

$$\frac{1-a}{a-a^2} = a^2 \text{———} (B)$$

subs. (A) & (B) in eqn (5)

$$\bar{A}^{1} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}$$

subs. $\bar{A}^1$ in $V_{sy} = \bar{A}^1 V$.

$$V_{sy} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} V.$$

subs. V_{sy}, V.

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \quad \text{——— (6)}$$

$$V_{a0} = \frac{1}{3}\left[V_a + V_b + V_c\right]$$

$$V_{a1} = \frac{1}{3}\left[V_a + aV_b + a^2V_c\right]$$

$$V_{a2} = \frac{1}{3}\left[V_a + a^2V_b + aV_c\right].$$

From eqn (6) the symmetrical components can be calculated from unbalanced voltages.

* Similarly unbalanced current vectors is gn. by,

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} \quad \text{——— (7).}$$

ie,

$$I_a = I_{a0} + I_{a1} + I_{a2}$$

$$I_b = I_{a0} + a^2 I_{a1} + a I_{a2}$$

$$I_c = I_{a0} + a I_{a1} + a^2 I_{a2}$$

Symmetrical components of unbalanced current vectors.

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \quad \text{———} (8)$$

ie,

$$I_{a0} = \frac{1}{3}\left[I_a + I_b + I_c\right]$$

$$I_{a1} = \frac{1}{3}\left[I_a + a I_b + a^2 I_c\right]$$

$$I_{a2} = \frac{1}{3}\left[I_a + a^2 I_b + a I_c\right]$$

1. The voltage across a 3-ϕ unbalanced load are $V_a = 300\angle 20^\circ V$, $V_b = 360\angle 90^\circ V$; $V_c = 500\angle -140^\circ V$. Determine the symmetrical components of voltages. phase sequence is abc.

To find :

V_{a0}, V_{b0}, V_{c0} ← Zero sequence components.

V_{a1}, V_{b1}, V_{c1} ← Positive sequence components.

V_{a2}, V_{b2}, V_{c2} ← Negative sequence components.

Given:

$V_a = 300\angle 20^\circ V.$

$= 281.91 + j102.61 V.$

$V_b = 360\angle 90^\circ V$

$= 0 + j360 V.$

$V_c = 500\angle -140^\circ V$

$= -383.02 - j321.39 V.$

$aV_b = 1\angle 120^\circ \times 360^\circ\angle 90^\circ$

$= 360\angle 210^\circ.$

$= -311.77 - j180 V.$

$a^2V_b = 1\angle 240^\circ \times 360^\circ\angle 90^\circ$

$= 360\angle 330^\circ$

$= 311.77 - j180 V.$

$aV_c = 1\angle 120^\circ \times 500\angle -140^\circ$

$= 500\angle -20^\circ.$

$= 469.85 - j171.01 V.$

$a^2V_c = 1\angle 240^\circ \times 500\angle -140^\circ$

$= 500\angle 100^\circ$

$= -86.82 + j492.4 V.$

$V_{a0} = \frac{1}{3}(V_a + V_b + V_c)$

$= \frac{1}{3}(281.91 + j102.61 + 0 + j360 - 383.02 - j321.39)$

$= \frac{1}{3}(-101.11 + j141.22)$

$= -33.70 + j47.07$

$= 57.89\angle 129^\circ$ V.

$V_{a1} = \frac{1}{3}[V_a + aV_b + a^2V_c]$

$= \frac{1}{3}[281.91 + j102.61 - 311.77 - j180 - 86.82 + j492.4]$

$= \frac{1}{3}[-116.68 + j415.01]$

$= -38.89 + j138.34$

$= 143.7\angle 106^\circ$ V.

$V_{a2} = \frac{1}{3}[V_a + a^2V_b + aV_c]$

$= \frac{1}{3}[281.91 + j102.61 + 311.77 - j180 + 469.85 - j171.01]$

$= \frac{1}{3}[1063.53 - j248.40]$

$= 354.51 - j82.80$

$= 364.05\angle -13^\circ$ V.

$V_{a0} = V_{b0} = V_{c0}$.

$\therefore \quad V_{a0} = 57.89\angle 126^\circ$ V.

$V_{b0} = 57.89\angle 126^\circ$ V.

$V_{c0} = 57.89\angle 126^\circ$ V.

$V_{b1} = a^2V_{a1}$

$V_{a1} = 143.70\angle 106^\circ$ V.

$V_{b1} = 1\angle 240^\circ \times 143.70\angle 106^\circ$ V.

$= 143.70\angle 346^\circ V.$

$V_{c1} = aV_{a1}$

$= 1\angle 120^\circ \times 143.70\angle 106^\circ.$

$= 143.70\angle 226^\circ V.$

$V_{b2} = aV_{a2}$; $V_{c2} = a^2V_{a2}$

$V_{a2} = 364.05\angle -13^\circ V.$

$V_{b2} = aV_{a2} = 1\angle 120^\circ \times 364.05\angle -13^\circ$

$= 364.05\angle 107^\circ V.$

$V_{c2} = a^2V_{a2} = 1\angle 240^\circ \times 364.05\angle -13^\circ.$

$= 364.05\angle 227^\circ V.$

The symmetrical components of phase-a voltage in a 3-ϕ unbalanced system are $V_{a0} = 10\angle 180^\circ V$, $V_{a1} = 50\angle 0^\circ V$ and $V_{a2} = 20\angle 90^\circ$. Determine phase voltages V_a, V_b, V_c.

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}$$

$V_a = V_{a0} + V_{a1} + V_{a2}$

$V_b = V_{a0} + a^2V_{a1} + aV_{a2}$

$V_c = V_{a0} + aV_{a1} + a^2V_{a2}$

given,

$$V_{a0} = 10\angle 180^\circ V = -10 + j0.$$

$$V_{a1} = 50\angle 0^\circ V = 50 + j0$$

$$V_{a2} = 20\angle 90^\circ V = 0 + j20.$$

$$aV_{a1} = 1\angle 120^\circ \times 50 \times 0^\circ = 5\angle 120^\circ = -25 + j43.30.$$

$$a^2 V_{a1} = 1\angle 240^\circ \times 50\angle 0^\circ = 50\angle 240^\circ = -25 - j43.30$$

$$aV_{a2} = 1\angle 120^\circ \times 20\angle 90^\circ = 20\angle 210^\circ = -17.32 - j10$$

$$a^2 V_{a2} = 1\angle 240^\circ \times 20\angle 90^\circ = 20\angle 330^\circ = 17.32 - j10.$$

$$V_a = V_{a0} + V_{a1} + V_{a2}$$

$$= -10 + 50 + j20$$

$$= 40 + j20$$

$$= 44.72\angle 27^\circ V.$$

$$V_b = V_{a0} + a^2 V_{a1} + aV_{a2}$$

$$= -10 - 25 - j43.30 - 17.32 - j10$$

$$= -52.32 - j53.3$$

$$= 74.69\angle -134^\circ V.$$

$$V_c = V_{a0} + aV_{a1} + a^2 V_{a2}$$

$$= -10 - 25 + j43.3 + 17.32 - j10.$$

$$= -17.68 + j33.3$$

$$= 37.7\angle 118^\circ V.$$

Sequence Impedances:

The impedances offered by the circuit elements to positive, negative and zero sequence currents is called sequence impedances.

* Positive sequence impedance.
* Negative sequence impedance.
* Zero sequence impedance.

Positive Sequence Impedance:

The impedance of a circuit element when positive sequence currents alone are flowing is called the positive sequence impedance.

Negative Sequence Impedance:

The impedance of a circuit element when negative sequence currents alone are flowing is called the negative sequence impedance.

Zero Sequence impedance:

The impedance of a circuit element when zero sequence currents alone are flowing is called the zero sequence impedance.

Sequence Network:

The impedance or reactance diagram formed using the impedance or reactance of any one sequence only is called the sequence network for that particular sequence.

* Positive sequence Network.
* Negative sequence Network.
* Zero sequence Network.

Positive sequence Network:

The impedance (or) reactance diagram formed used positive sequence impedance is called positive sequence network.

Negative sequence Network:

The impedance (or) reactance diagram formed using positive sequence impedance or reactance is called Negative sequence network.

Zero sequence Network:

The impedance (or) reactance diagram formed using zero sequence impedance (or) reactance is called zero sequence network

* The sequence impedances and networks are useful in the analysis of unsymmetrical faults in the power system.
* In unsymmetrical fault analysis of a power system, the positive negative and zero sequence networks of the system are determined and then they are interconnected to represent various unbalanced fault conditions.

* Generator.
* Transmission line.
* Transformer.
* Loads.

} Sequence Networks.

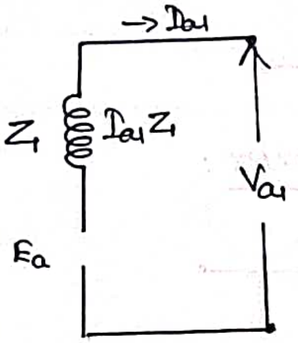
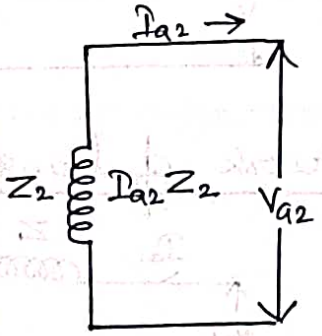
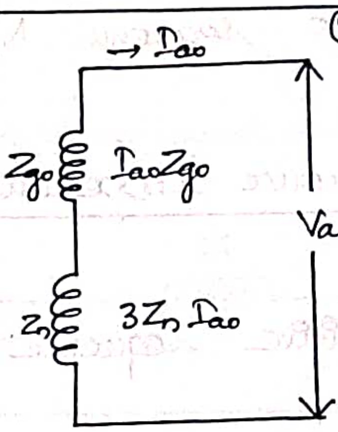
Sequence Impedance and Networks of Generator

E_a, E_b, E_c — Generated emf per phase in phase a, b and c respectively.

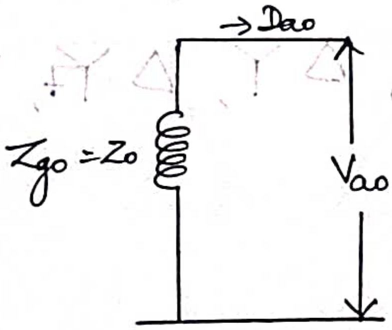
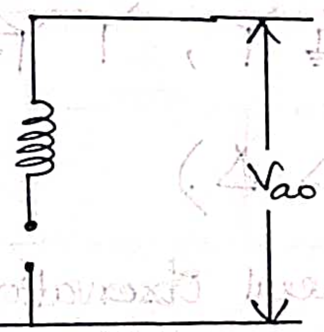
Z_1 – positive sequence impedance.

Z_2 – Negative sequence impedance.

Z_0 – Zero sequence impedance (total).

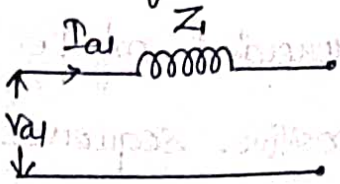
Positive Sequence N/w	Negative Sequence N/w	Zero sequence network. ① reactance grounding
		
$V_{a1} = E_a - I_{a1}Z_1$	$V_{a2} = -I_{a2}Z_2$	$V_{a0} = -3I_{a0}Z_n - I_{a0}Z_{g0}$. $= -I_{a0}(3Z_n + Z_{g0})$ $= -I_{a0}Z_0$.

Zero sequence network:

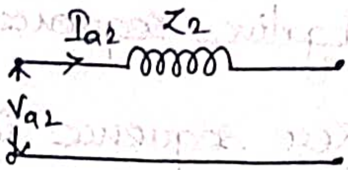
② Solid grounding	③ No grounding.
	

Sequence Impedances & Network of transmission lines

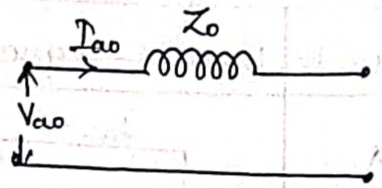
Positive sequence network



Negative Sequence Network

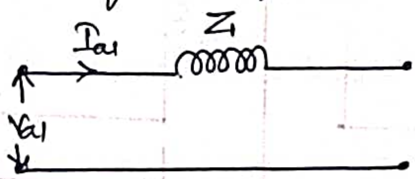


Zero sequence Network

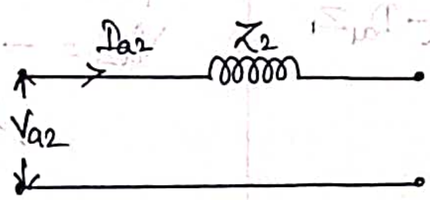


Sequence Impedances & Network of transformer:

Positive sequence Network



Negative Sequence Network

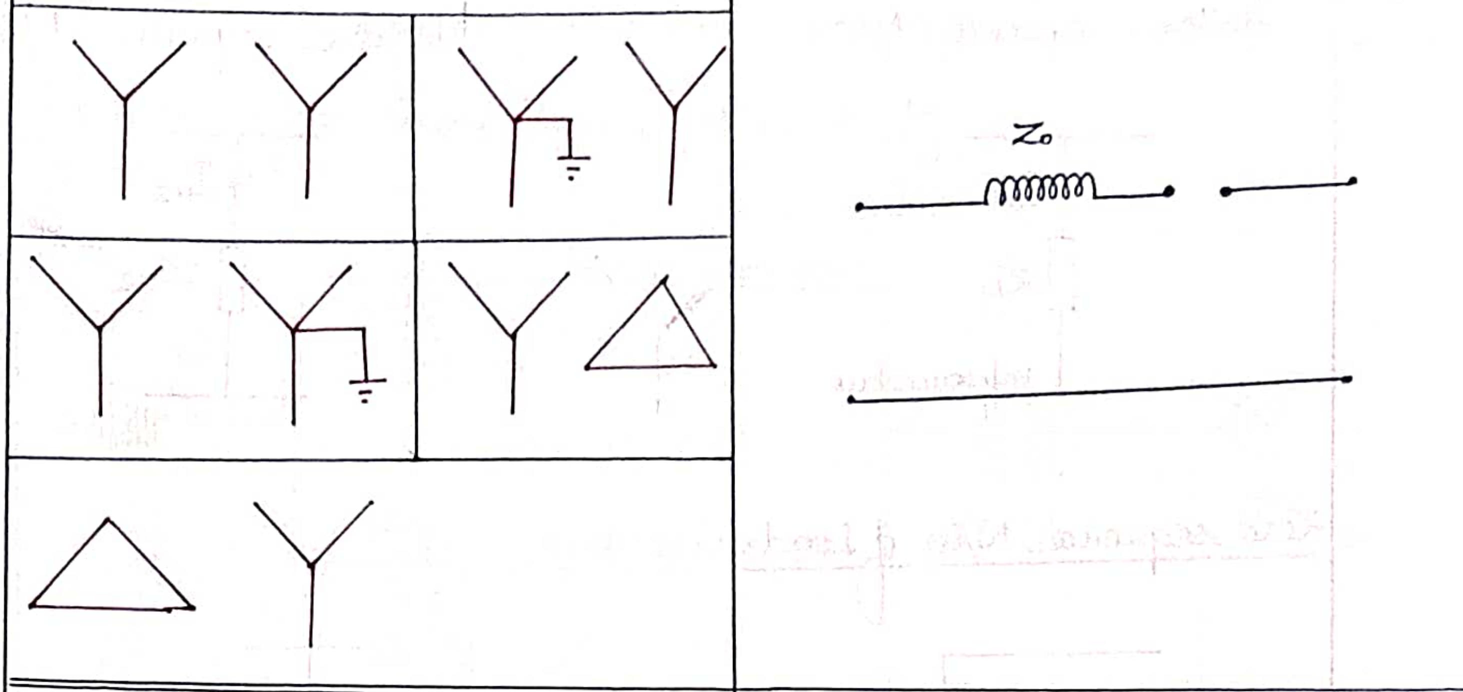


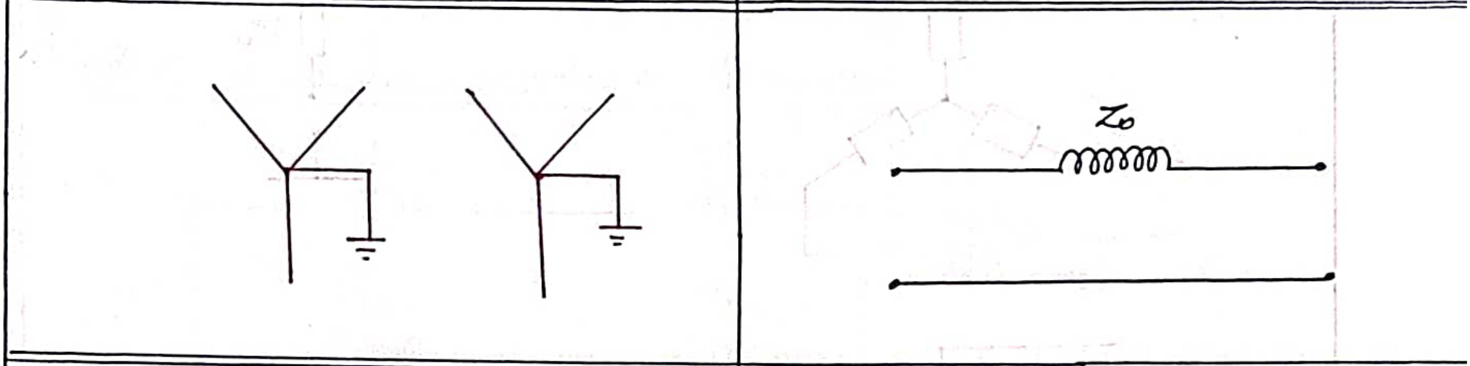
* The zero sequence n/w depends on Y or Δ connection.

* 9 types of configurations are possible, they are (Y Y, Y⏚ Y, Y Y⏚, Y⏚ Y⏚, Y Δ, Δ Y, Δ Y⏚, Y⏚ Δ, Δ Δ)

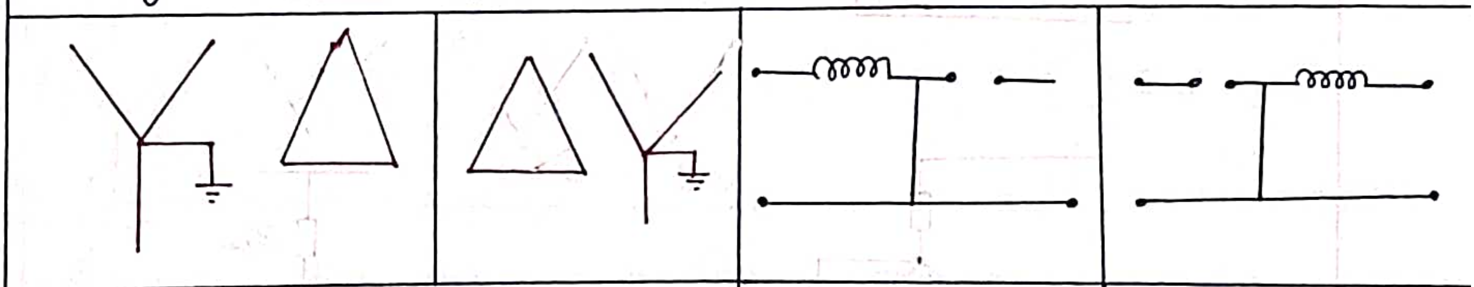
General Observations are,

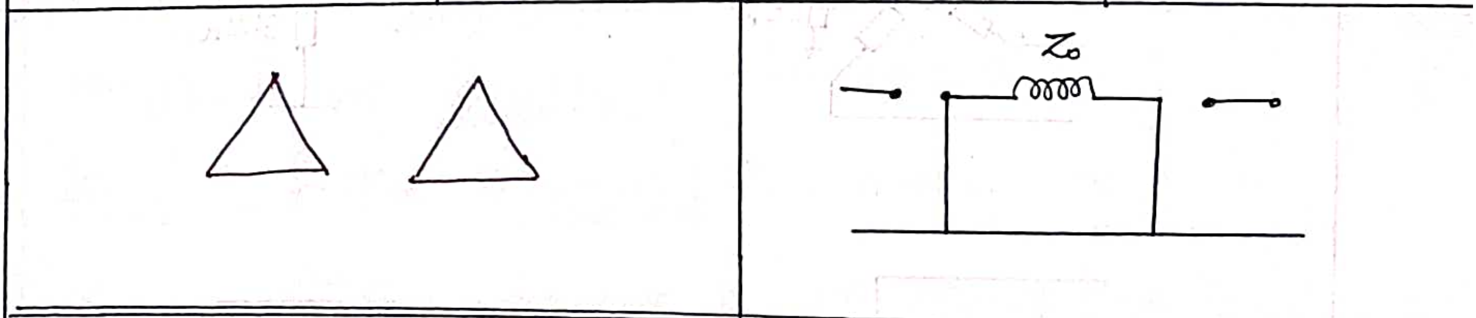
* If the neutral point in the Y connected winding is not grounded → no zero sequence current,





*The zero sequence current can circulate in the delta connected winding but cannot flow through the lines.

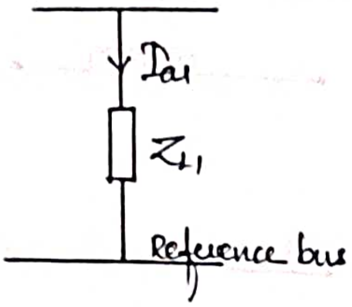




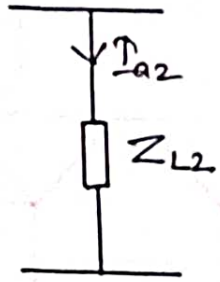
Sequence impedances & networks of loads:

The positive and negative sequence impedances of load are represented as a shunt impedance.

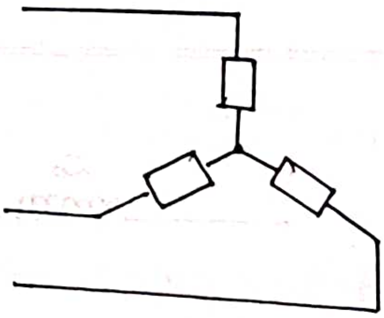
Positive Sequence N/w

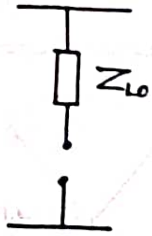


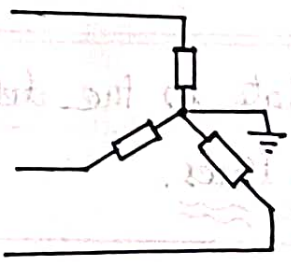
Negative Sequence N/w.

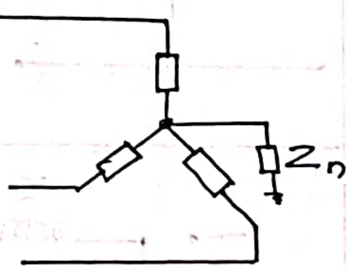


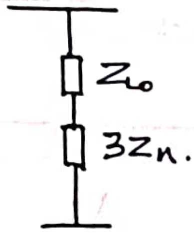
Zero sequence N/w of Loads:

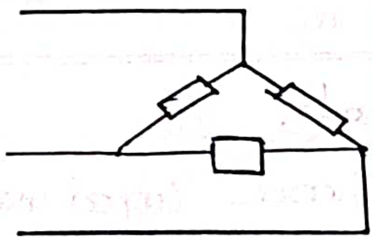


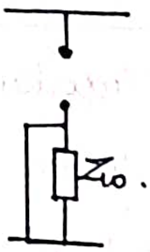




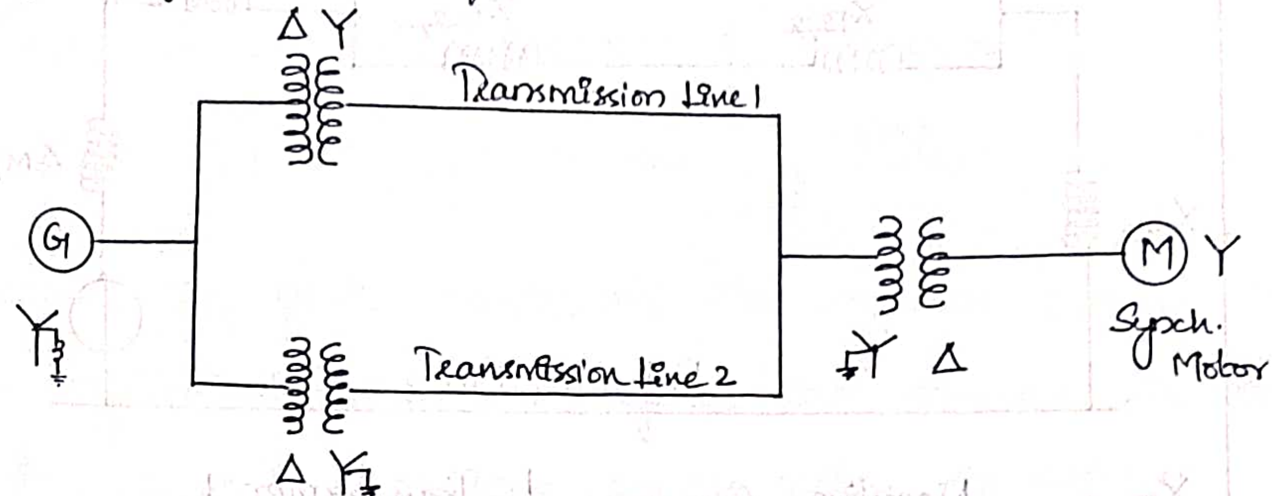




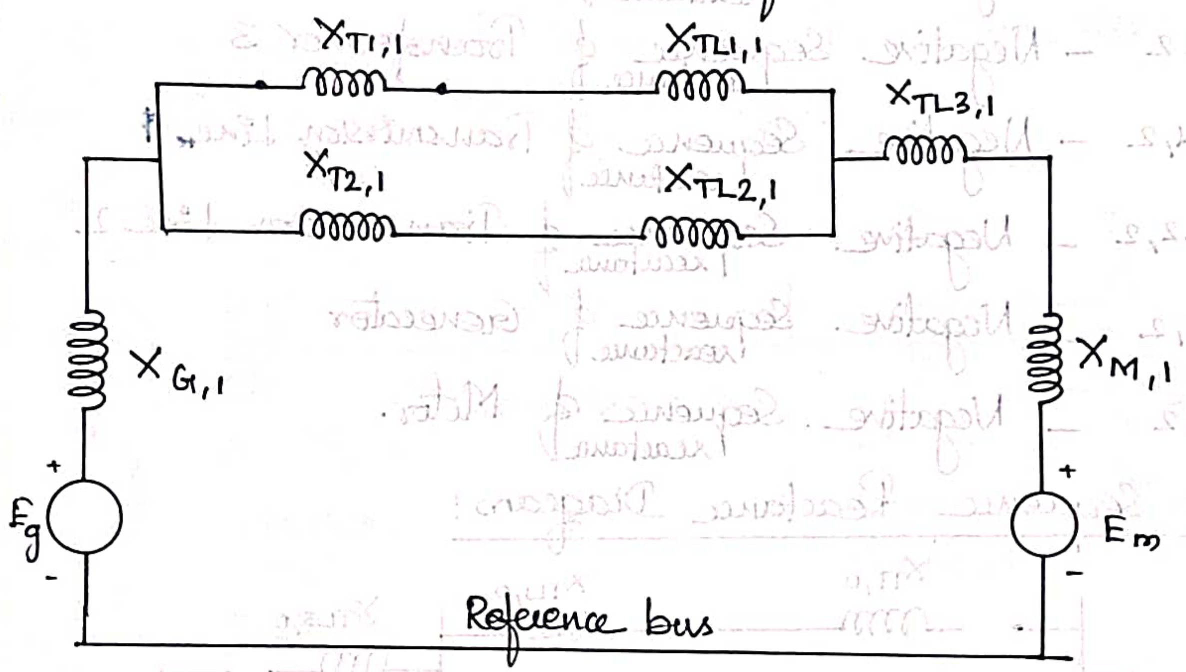




1. Draw the positive, negative and zero sequence reactance diagrams of the power system shown in fig.



Positive Sequence reactance diagram



$X_{T1,1}$ – Positive Sequence reactance of Transformer 1.

$X_{T2,1}$ – Positive Sequence reactance of Transformer 2.

$X_{TL1,1}$ – Positive Sequence reactance of Transmission Line 1.

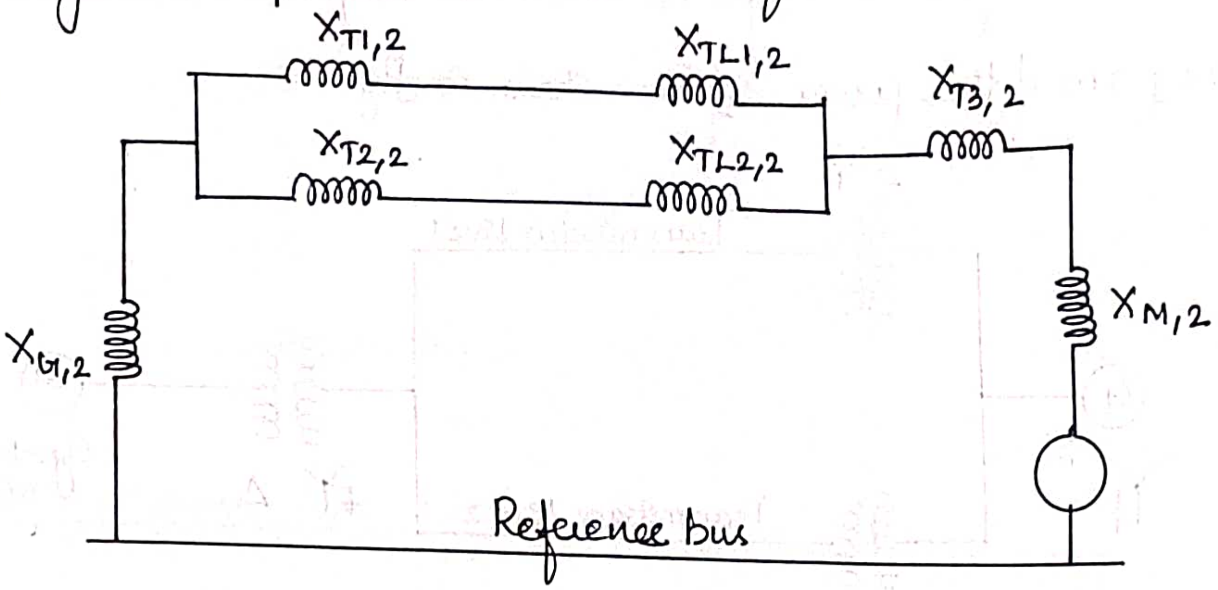
$X_{TL2,1}$ – Positive Sequence reactance of Transmission Line 2.

$X_{T3,1}$ – Positive Sequence reactance of Transformer 3.

$X_{G,1}$ – Positive Sequence reactance of Generator 1

$X_{M,1}$ – Positive Sequence reactance of Motor 1.

Negative Sequence Reactance diagram:



$X_{T1,2}$ – Negative Sequence reactance of Transformer 1

$X_{T2,2}$ – Negative Sequence reactance of Transformer 2

$X_{T3,2}$ – Negative Sequence reactance of Transformer 3

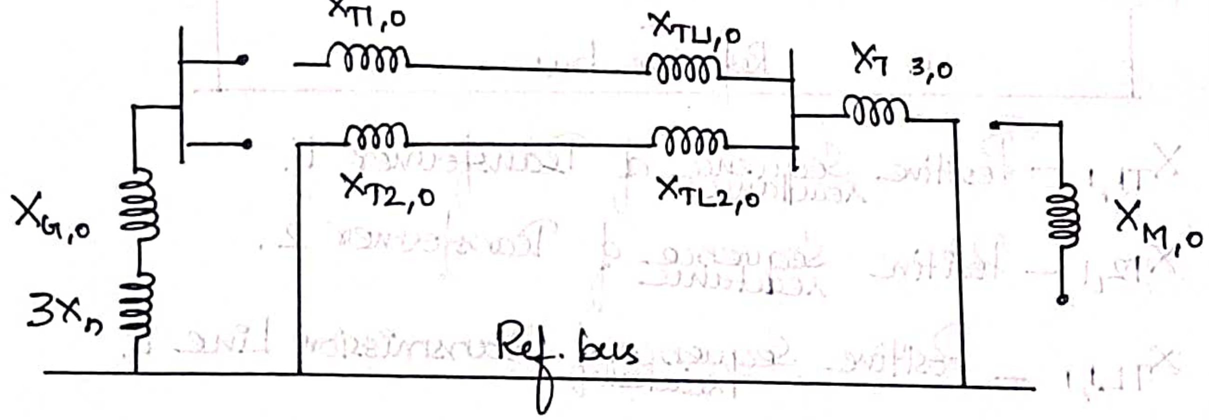
$X_{TL1,2}$ – Negative Sequence reactance of Transmission Line 1

$X_{TL2,2}$ – Negative Sequence reactance of Transmission line 2

$X_{G,2}$ – Negative Sequence reactance of Generator

$X_{M,2}$ – Negative Sequence reactance of Motor.

Zero Sequence Reactance Diagram:



$X_{T1,0}$ – Zero Sequence reactance of Transformer 1.

$X_{T2,0}$ – Zero Sequence reactance of Transformer 2.

$X_{T3,0}$ – Zero Sequence reactance of Transformer 3.

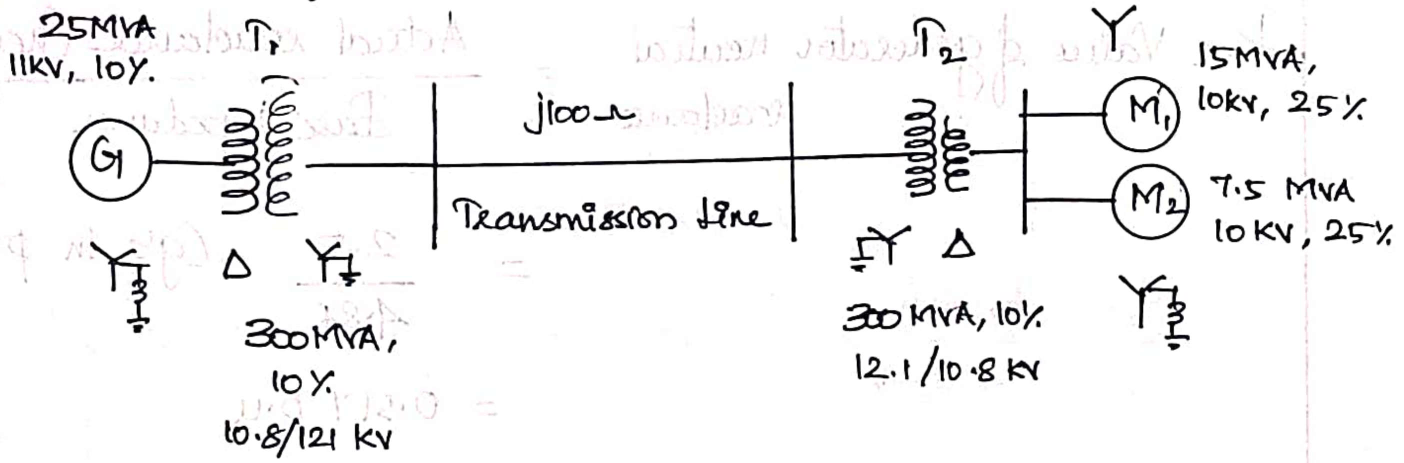
$X_{TL1,0}$ – Zero Sequence reactance of Transmission Line 1

$X_{TL2,0}$ – Zero Sequence reactance of Transmission Line 2.

$X_{G1,0}$ – Zero sequence reactance of Generator

$X_{M,0}$ – Zero sequence reactance of Motor.

2. Determine the positive, negative and zero sequence networks for the system shown in fig. Assume zero sequence reactances for the generator and synchronous motors as 0.06 p.u. Current limiting reactors of 2.5 Ω are connected in the neutral of the generator and motor no. 2. The zero sequence reactance of the T. Line is j300 Ω.



Base Values:

$MVA_{b,new} = 25 MVA$

$KV_{b,new} = 11 KV$,

$X_{G,0} = 0.06$ p.u.

$X_{M,0} = 0.06$ p.u.

$X_{TL,0} = j300$ p.u.

Sequence Reactance of Generator G:

Positive sequence reactance, $X_{G,1}$ = 0.1 p.u

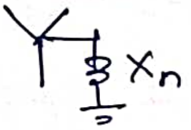
Negative sequence reactance $X_{G,2}$ = 0.1 p.u.

} Since base values of generator rating are same, the reactance does not change.

Zero sequence reactance, $X_{G,0}$ = 0.06 p.u

$$\text{Base impedance}, Z_b = \frac{(KV_{b,new})^2}{MVA_{b,new}}$$

$$= \frac{11^2}{25}$$



$$= 4.84\,\Omega.$$

$$\text{P.u value of generator neutral reactance} = \frac{\text{Actual reactance (neutral)}}{\text{Base impedance}}$$

$$= \frac{2.5}{4.84} \quad (\text{g.v. in prob})$$

$$= 0.517 \text{ p.u}$$

Sequence reactance of Transformer T_1:

$$X_{p.u,new} \text{ of } T_1 = X_{p.u,old} \times \left[\frac{KV_{b,old}}{KV_{b,new}}\right]^2 \times \left[\frac{MVA_{b,new}}{MVA_{b,old}}\right]$$

$$= 0.1 \times \left[\frac{10.8}{11}\right]^2 \times \left[\frac{25}{30}\right]$$

$$= 0.08 \text{ p.u.}$$

Assume,

Positive Sequence reactance of transformer T_1, $X_{T1,1} = 0.08$ p.u

Negative Sequence reactance of transformer T_1 $X_{T1,2} = 0.08$ p.u

Zero sequence reactance of transformer T_1, $X_{T1,0} = 0.08$ p.u

Sequence Reactance of Transmission Line:

$$\text{Base kV on HT side} = \text{Base kV on LT side} \times \frac{\text{HT voltage rating}}{\text{LT voltage rating}}$$

$$= 11 \times \frac{121}{10.8}$$

$$KV_{b,new} = 123.24 \text{ kV.}$$

$$Z_b = \frac{(KV_b)^2}{MVA_b}$$

$$= \frac{123.24^2}{30}$$

$$= 506.27\,\Omega.$$

$$\text{P.u reactance of transmission line} = \frac{\text{Actual reactance}}{\text{Base impedance}}$$

$$= \frac{100}{506.27} \quad (\text{gv. in prob})$$

$$= 0.198 \text{ p.u}$$

Positive & Negative Sequence reactances, $X_{TL,1} = 0.198$ p.u.

$X_{TL,2} = 0.198$ p.u

Zero sequence reactance of transmission line, $= j300\,\Omega$.

ie, $X_{TL,0} = \frac{\text{Zero sequence reactance } (\Omega)}{\text{Base impedance}, Z_b}$

$= \frac{300}{506.27}$

$X_{TL,0} = 0.593 \text{ p.u.}$

Sequence reactances of Transformer, T_2.

$X_{p.u,new} = X_{p.u,old} \times \left[\frac{KV_{b,old}}{KV_{b,new}}\right]^2 \times \left[\frac{MVA_{b,new}}{MVA_{b,old}}\right]$.

$= 0.1 \times \left[\frac{10.8}{11}\right]^2 \times \left[\frac{25}{30}\right]$

$= 0.08 \text{ p.u}$

(or) same ratings of transformer.

$X_{T2,1} = X_{T1,1} = 0.08 \text{ p.u.}$

$X_{T2,2} = X_{T1,1} = 0.08 \text{ p.u.}$

$X_{T2,0} = X_{T1,0} = 0.08 \text{ p.u.}$

Sequence reactances of Synchronous Motor, M,

Base KV on LT side on transformer T_2 $= \text{Base KV on HT side} \times \frac{\text{LT Voltage ratio}}{\text{HT Voltage ratio}}$

$= 123.24 \times \frac{10.8}{121}$

$= 11 KV.$

$$X_{pu,new} = X_{p.u,old} \times \left[\frac{KV_{b,old}}{KV_{b,new}}\right]^2 \times \left[\frac{MVA_{b,new}}{MVA_{b,old}}\right]$$

$$= 0.25 \times \left[\frac{10}{11}\right]^2 \times \left[\frac{25}{15}\right]$$

$$= 0.344 \text{ p.u}$$

$$X_{M1,1} = 0.344 \text{ p.u}$$

$$X_{M1,2} = 0.344 \text{ p.u}$$

0 Sequence reactance, $= 0.06$ (gv. in prob)

$$X_{M1,0} = X_{p.u,old} \times \left[\frac{KV_{b,old}}{KV_{b,new}}\right]^2 \times \left[\frac{MVA_{b,new}}{MVA_{b,old}}\right]$$

$$= 0.06 \times \left(\frac{10}{11}\right)^2 \times \left(\frac{25}{15}\right)$$

$$= 0.083 \text{ p.u.}$$

Sequence reactances of Synchronous Motor M_2:

$MVA_{b,old} = 7.5$ MVA $\qquad X = 0.25$

$KV_{b,old} = 10$ kV

$$X_{p.u,new} = X_{M2,1} = 0.25 \times \left[\frac{10}{11}\right]^2 \times \left[\frac{25}{7.5}\right]$$

$$= 0.689 \text{ p.u.}$$

lly.

$$X_{M2,2} = 0.689 \text{ p.u.}$$

zero sequence reactance $= 0.06$ p.u (gv. in prob).

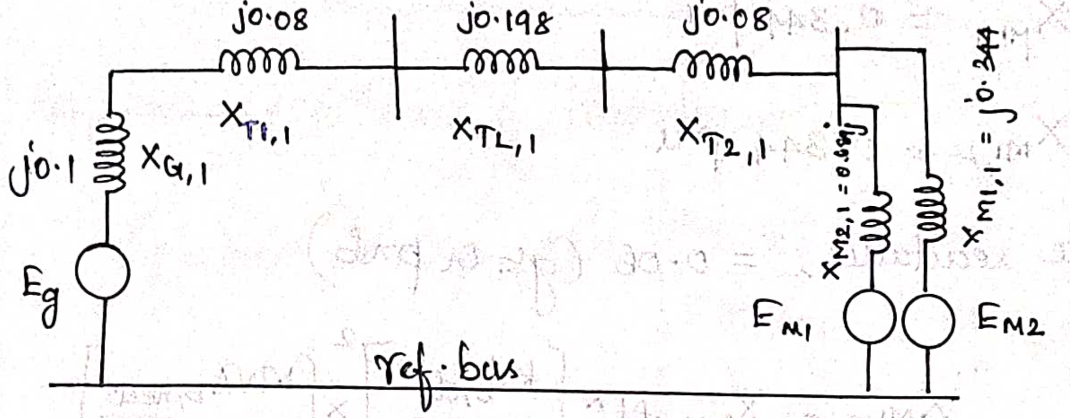
$$X_{pu,new} = X_{M2,0} = 0.06 \times \left[\frac{10}{11}\right]^2 \left[\frac{25}{7.5}\right] = 0.165 \text{ pu}$$

Motor is grounded through reactance, so,

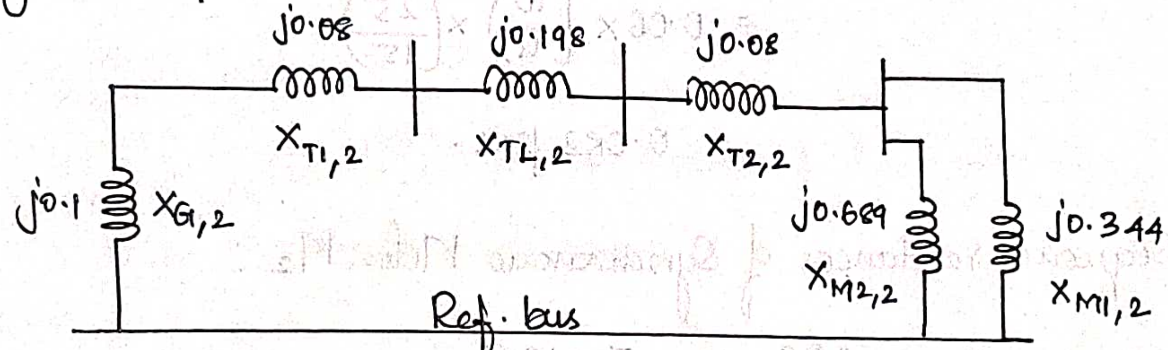
$$Z_b = \frac{11^2}{25} = 4.84\,\Omega.$$

p.u Value of Motor neutral reactance } $X_{MN} = \frac{\text{Actual neutral reactance}}{\text{Base impedance}} = \frac{2.5}{4.84} = 0.51$

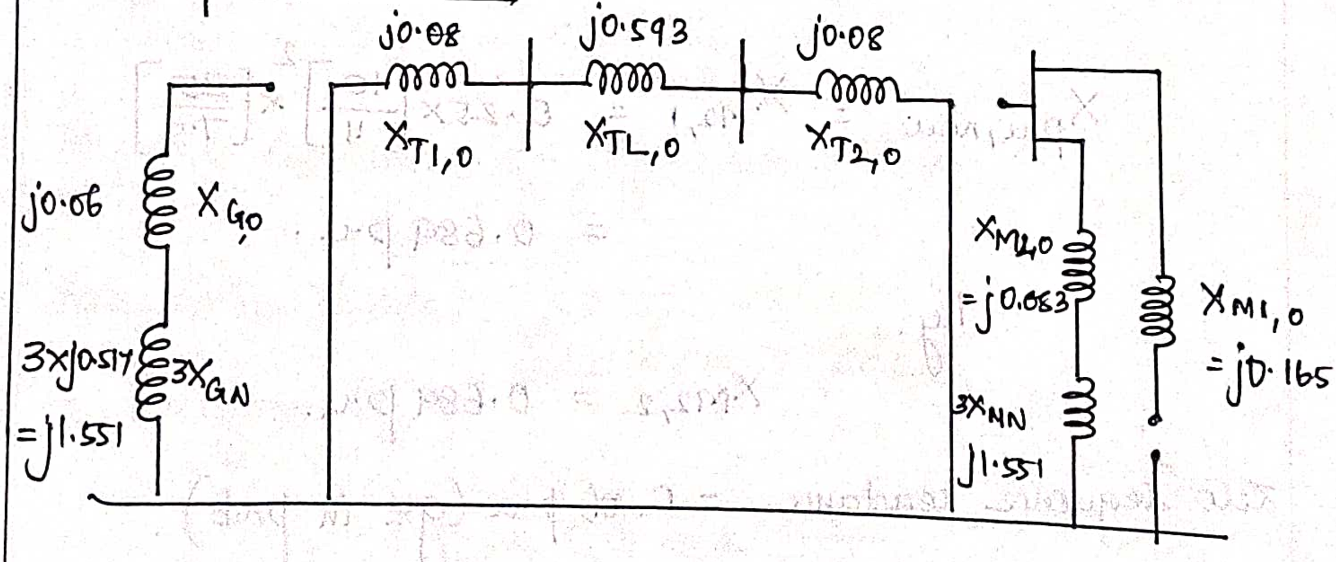
Positive Sequence Network



Negative Sequence Network



Zero Sequence Network :



Unsymmetrical Fault:

If the faults currents in the three phases are unequal, then it is called unsymmetrical faults.

i) Single-Line to ground fault.

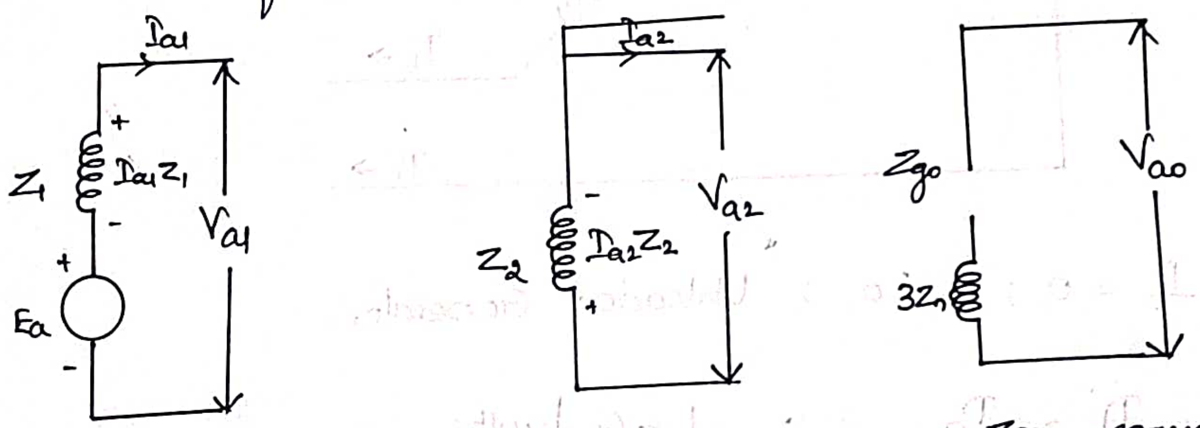
ii) Line-Line fault.

iii) Double line to ground fault.

* The unsymmetrical faults are analysed using symmetrical components.

Sequence Network Equation of Generator:

The positive, negative and zero sequence networks of generator with neutral grounded through an impedance Z_n are shown.



Positive sequence n/w of a generator

Negative sequence network of a generator

Zero sequence n/w of generator.

$$V_{a0} = -I_{a0}Z_0 \quad \text{---(1)}$$

$$V_{a1} = E_a - I_{a1}Z_1 \quad \text{---(2)}$$

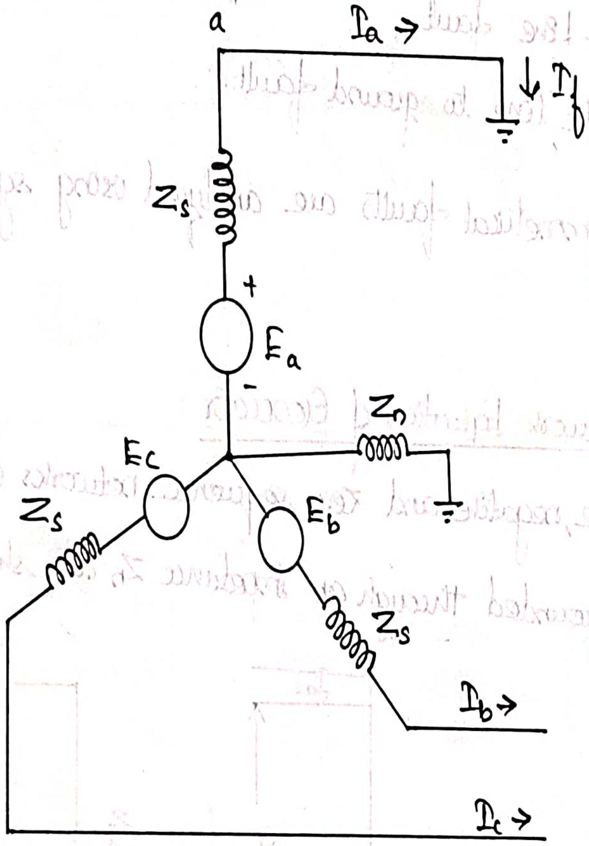
$$V_{a2} = -I_{a2}Z_2 \quad \text{---(3)}$$

From (1), (2) & (3),

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} \quad \text{---(4)}$$

The above equation is the sequence n/o equation of generator & is used in unsymmetrical fault analysis on the generator.

I. Single Line to Ground fault on an unloaded Generator:



$I_b = 0$; $I_c = 0$; 'Unloaded' Generator.

$I_f = I_a$; L-G fault.

$V_a = 0$; Faulty phase.

Step:1 $I_b = 0$; $I_c = 0$; subs in symmetrical components of current

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a^2 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$I_{a0} = \frac{1}{3} I_a$$

$$I_{a1} = \frac{1}{3} I_a$$

$$I_{a2} = \frac{1}{3} I_a$$

i.e. $I_{a0} = I_{a1} = I_{a2} = \frac{1}{3} I_a$ ——— (1)

step:2 Subs (1) in sequence n/w equation of generator,

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} \begin{bmatrix} I_{a1} \\ I_{a1} \\ I_{a1} \end{bmatrix} \quad \text{——— (2)}$$

$$V_{a0} = -Z_0 I_{a1} \quad \text{——— (i)}$$

$$V_{a1} = E_a - Z_1 I_{a1} \quad \text{——— (ii)}$$

$$V_{a2} = -Z_2 I_{a1} \quad \text{——— (iii)}$$

step:3 $V_a = 0$

WKT, $V_a = V_{a0} + V_{a1} + V_{a2} = 0$

$$V_a = -Z_0 I_{a1} + E_a - Z_1 I_{a1} - Z_2 I_{a1} = 0.$$

$$E_a = Z_0 I_{a1} + Z_1 I_{a1} + Z_2 I_{a2}$$

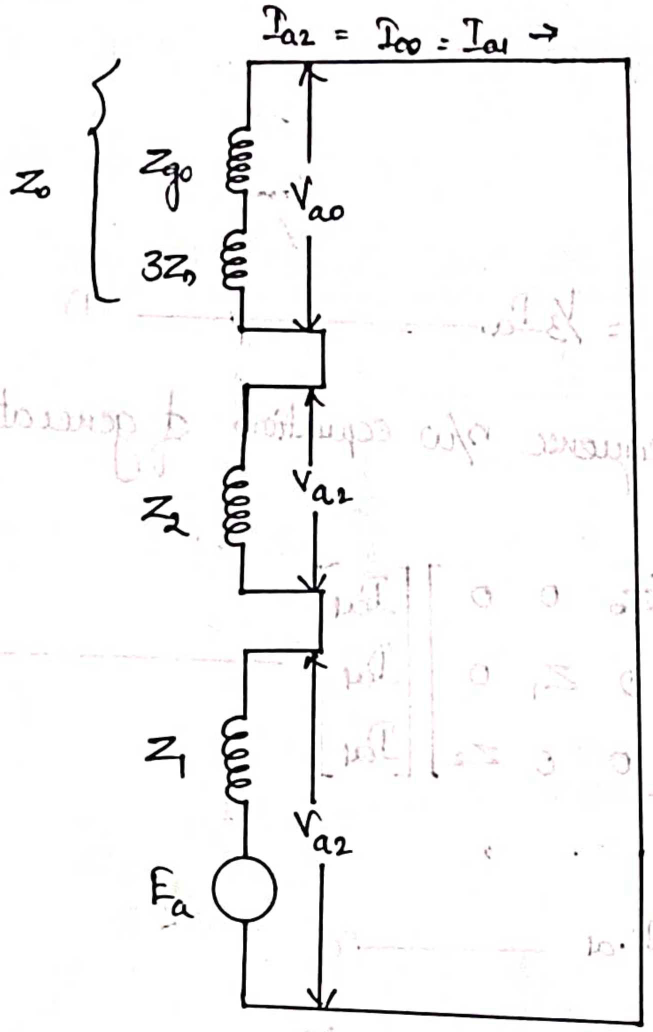
I_{a1} (

$$\boxed{I_{a1} = \frac{E_a}{Z_0 + Z_1 + Z_2}} \quad \text{——— (3)}$$

step:4 Fault Current:

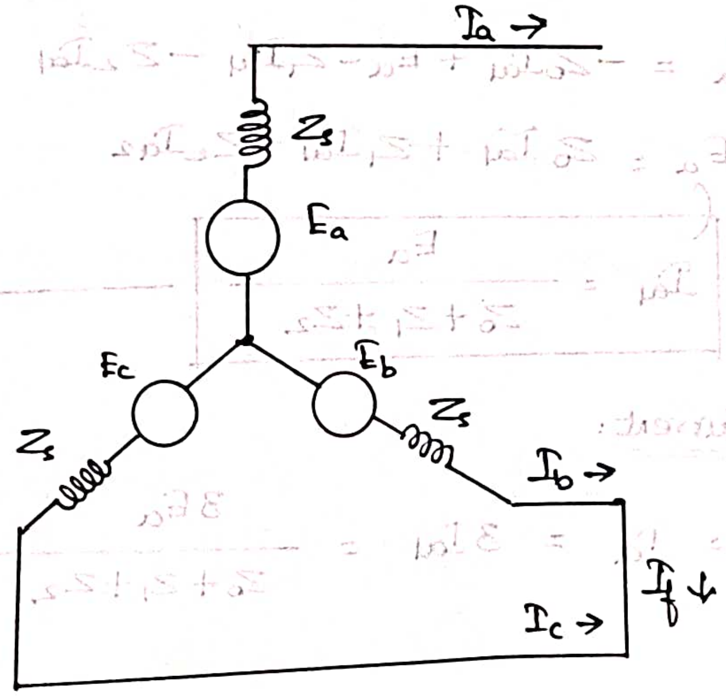
$$I_f = I_a = 3 I_{a1} = \frac{3 E_a}{Z_0 + Z_1 + Z_2}$$

Step : 5 Equivalent Circuit:



If neutral is not grounded, then the circuit will be open & no current flows. $I_{a0} = I_{a1} = I_{a2} = 0$.

II. Line To Line Fault on an unloaded Generator:



Step :1 $V_b = V_c$ subs in symmetrical components of voltage Eqn.

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_b \end{bmatrix} \quad \text{——— (1)}$$

$$V_{a0} = \tfrac{1}{3}[V_a + V_b + V_b]$$

$$= \tfrac{1}{3}[V_a + 2V_b].$$

$$V_{a1} = \tfrac{1}{3}[V_a + (a + a^2)V_b] \quad \text{——— (2)}$$

$$V_{a2} = \tfrac{1}{3}[V_a + (a + a^2)V_b] \quad \text{——— (3)}$$

From Eqn. (2) & (3)

$$\boxed{V_{a1} = V_{a2}}$$

Step :2 $I_a = 0$; open (ie no loaded Generator)

$$I_b = -I_c.$$

subs above said in symmetrical components of current Eqn.

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} 0 \\ -I_c \\ I_c \end{bmatrix} \quad \text{——— (4).}$$

$$I_{a0} = \tfrac{1}{3}[-I_c + I_c] = 0$$

$$\boxed{I_{a0} = 0} \quad \text{——— (4.1)}$$

$$I_{a1} = \frac{1}{3}[-aI_c + a^2 I_c]$$

$$I_{a1} = \frac{1}{3}[I_c(a^2 - a)] \quad \text{——— (5)}$$

$$I_{a2} = \frac{1}{3}[-a^2 I_c + aI_c]$$

$$I_{a2} = -\frac{1}{3}[I_c(a^2 - a)] \quad \text{——— (6)}$$

From (5) & (6)

$$I_{a1} = -I_{a2} \quad \text{——— (7)}$$

Step : 3 Subs Eqn. (4.1) & (7) in sequence n/w equation of generator.

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} \begin{bmatrix} 0 \\ I_{a1} \\ -I_{a1} \end{bmatrix}$$

$$\boxed{\begin{aligned} V_{a0} &= 0 \\ V_{a1} &= E_a - Z_1 I_{a1} \\ V_{a2} &= Z_2 I_{a1} \end{aligned}} \quad \text{——— (8)}$$

Step : 4 WKT, $V_{a1} = V_{a2}$.

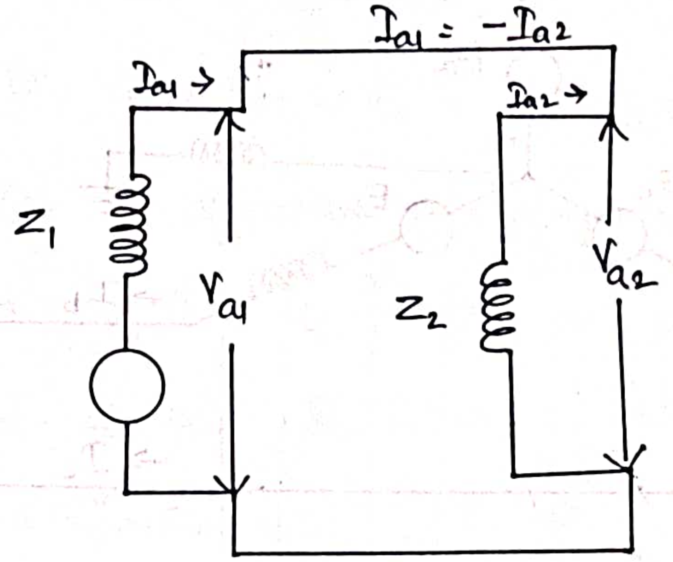
$$E_a - Z_1 I_{a1} = Z_2 I_{a1}$$

$$E_a = Z_2 I_{a1} + Z_1 I_{a1}$$

$$\boxed{I_{a1} = \frac{E_a}{Z_1 + Z_2}} \quad \text{——— (9)}$$

Step :5 Equivalent Circuit :

* Since $V_{a0} = 0$ & Z_0 does not enter into the equations, the zero sequence network is not used.



Step :6 Fault Current :

$$I_f = I_b = -I_c$$

$$I_b = I_{a0} + a^2 I_{a1} + a I_{a2} \quad \text{——— (10)}$$

KKT, $I_{a0} = 0$

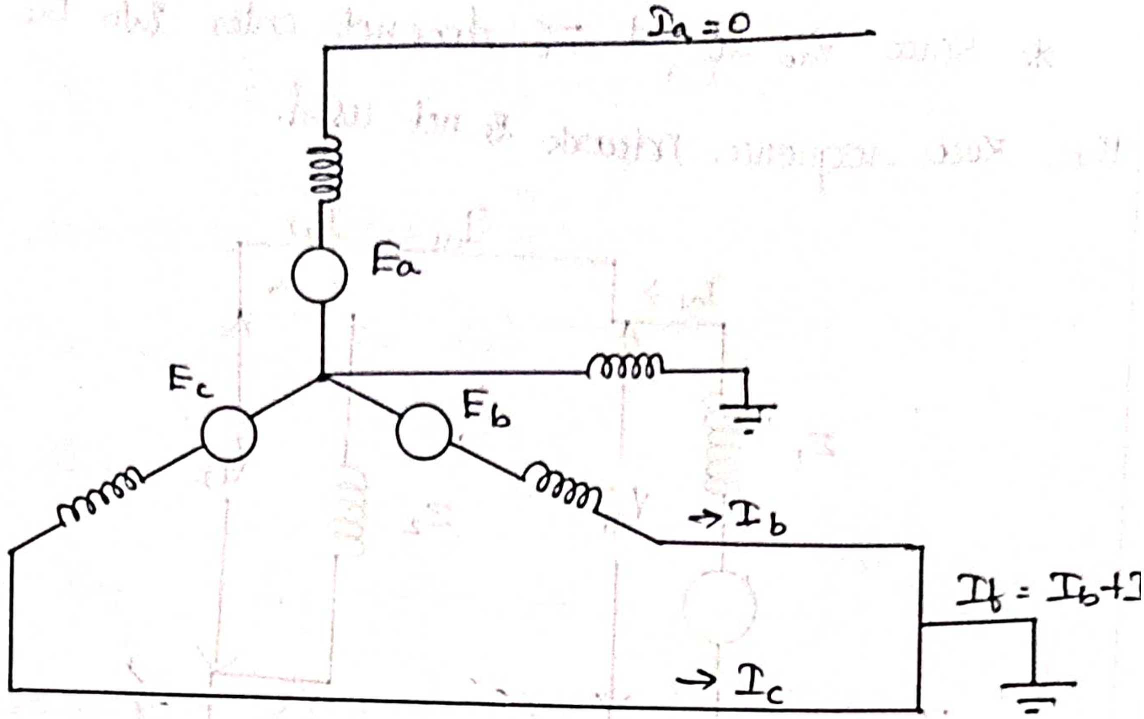
$$I_{a2} = -I_{a1}$$

Therefore (10) becomes,

$$I_b = a^2 I_{a1} + a(-I_{a1})$$

$$\boxed{I_f = I_b = I_{a1}(a^2 - a)} \quad \text{——— (11)}$$

III ※ Double Line to Ground Fault on an unloaded Genera[tor]



Step 1: $V_b = V_c = 0$ subs in symmetrical components of Voltage eqⁿ.

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} V_a \\ 0 \\ 0 \end{bmatrix}$$

$$V_{a0} = \frac{1}{3} V_a$$

$$V_{a1} = \frac{1}{3} V_a$$

$$V_{a2} = \frac{1}{3} V_a$$

ie $\boxed{V_{a0} = V_{a1} = V_{a2} = V_a/3}$ ——— (1)

Step: 2 Sequence n/w of generator Equation.

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix}\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$

From above eq^n.,

$$V_{a1} = E_a - I_{a1} Z_1 \quad \text{———— (2)}$$

From eq^n. (1) $V_{a1} = V_{a2} = V_{a0}$

$$\text{i.e.} \quad V_{a1} = V_{a2} = V_{a0} = E_a - I_{a1} Z_1 \quad \text{———— (3)}$$

From eq^n. (3) in sequence n/w of generator

$$\begin{bmatrix} E_a - I_{a1} Z_1 \\ E_a - I_{a1} Z_1 \\ E_a - I_{a1} Z_1 \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$

Re-arranging above eq^n.,

$$\begin{vmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{vmatrix} \begin{vmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{vmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} E_a - I_{a1} Z_1 \\ E_a - I_{a1} Z_1 \\ E_a - I_{a1} Z_1 \end{bmatrix} \quad \text{———— (4)}$$

p:3 Find Z^{-1}.

$$Z^{-1} = \frac{Z_{cof}^{T}}{\text{Determinant of } Z}$$

$$\Delta_z = \begin{vmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{vmatrix} = Z_0 Z_1 Z_2$$

$$\text{Cofactor of } Z_{11} = (+)\begin{vmatrix} Z_1 & 0 \\ 0 & Z_2 \end{vmatrix} = Z_1 Z_2$$

$$\text{Cofactor of } Z_{12} = (-)\begin{vmatrix} 0 & 0 \\ 0 & Z_2 \end{vmatrix} = 0.$$

$$\text{Cofactor of } Z_{13} = +\begin{vmatrix} 0 & Z_1 \\ 0 & 0 \end{vmatrix} = 0.$$

Cofactor of $Z_{21} = -\begin{vmatrix} 0 & 0 \\ 0 & Z_2 \end{vmatrix} = 0$

Cofactor of $Z_{22} = +\begin{vmatrix} Z_0 & 0 \\ 0 & Z_2 \end{vmatrix} = Z_0 Z_2$.

Cofactor of $Z_{23} = -\begin{vmatrix} Z_0 & 0 \\ 0 & 0 \end{vmatrix} = 0$

Cofactor of $Z_{31} = +\begin{vmatrix} 0 & 0 \\ Z_1 & 0 \end{vmatrix} = 0$

Cofactor of $Z_{32} = -\begin{vmatrix} Z_0 & 0 \\ 0 & 0 \end{vmatrix} = 0$

cofactor of $Z_{33} = +\begin{vmatrix} Z_0 & 0 \\ 0 & Z_1 \end{vmatrix} = Z_0 Z_1$

$$Z_{cof} = \begin{bmatrix} Z_1 Z_2 & 0 & 0 \\ 0 & Z_0 Z_2 & 0 \\ 0 & 0 & Z_0 Z_1 \end{bmatrix}$$

$$Z^{-1} = \frac{Z_{cof}}{\Delta_z}$$

$$= \frac{1}{Z_0 Z_1 Z_2} \begin{bmatrix} Z_1 Z_2 & 0 & 0 \\ 0 & Z_0 Z_2 & 0 \\ 0 & 0 & Z_0 Z_1 \end{bmatrix}$$

$$Z^{-1} = \begin{bmatrix} 1/Z_0 & 0 & 0 \\ 0 & 1/Z_1 & 0 \\ 0 & 0 & 1/Z_2 \end{bmatrix}$$

On premultiplying eqn. (4) by Z^{-1}, we get,

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \begin{bmatrix} 1/Z_0 & 0 & 0 \\ 0 & 1/Z_1 & 0 \\ 0 & 0 & 1/Z_2 \end{bmatrix} \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} 1/Z_0 & 0 & 0 \\ 0 & 1/Z_1 & 0 \\ 0 & 0 & 1/Z_2 \end{bmatrix} \begin{bmatrix} E_a - I_{a1} Z_1 \\ E_a - I_{a1} Z_1 \\ E_a - I_{a1} Z \end{bmatrix}$$

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \begin{bmatrix} 0 \\ E_a/Z_1 \\ 0 \end{bmatrix} - \begin{bmatrix} \frac{E_a - I_{a1}Z_1}{Z_0} \\ \frac{E_a - I_{a1}Z_1}{Z_1} \\ \frac{E_a - I_{a1}Z_1}{Z_2} \end{bmatrix}$$

ie,

$$I_{a0} = -\left[\frac{E_a - I_{a1}}{Z_0}\right] = -\frac{E_a}{Z_0} + \frac{I_{a1}Z_1}{Z_0}$$

$$I_{a1} = \frac{E_a}{Z_1} - \left[\frac{E_a - I_{a1}Z_1}{Z_1}\right] = \frac{E_a}{Z_1} - \frac{E_a}{Z_1} + \frac{I_{a1}Z_1}{Z_1} = I_{a1}$$

$$I_{a2} = -\left[\frac{E_a - I_{a1}Z_1}{Z_2}\right] = -\frac{E_a}{Z_2} + \frac{I_{a1}Z_1}{Z_2}$$

step : 4 Find I_{a1}

$$I_a = 0$$

$$I_a = I_{a0} + I_{a1} + I_{a2} = 0$$

$$I_a = -\frac{E_a}{Z_0} + \frac{I_{a1}}{Z_0} + I_{a1} + \frac{E_a}{Z_2} + \frac{I_{a1}Z_1}{Z_2} = 0.$$

$$\frac{Z_1 I_{a1}}{Z_0} + I_{a1} + \frac{I_{a1}Z_1}{Z_2} = \frac{E_a}{Z_0} + \frac{E_a}{Z_2}$$

Taking I_{a1} & E_a common,

$$I_{a1}\left[\frac{Z_1}{Z_0} + 1 + \frac{Z_1}{Z_2}\right] = E_a\left[\frac{1}{Z_0} + \frac{1}{Z_2}\right]$$

$$I_{a1}\left[Z_1\left(\frac{1}{Z_0} + \frac{1}{Z_2}\right) + 1\right] = E_a\left[\frac{Z_0 + Z_2}{Z_0 Z_2}\right]$$

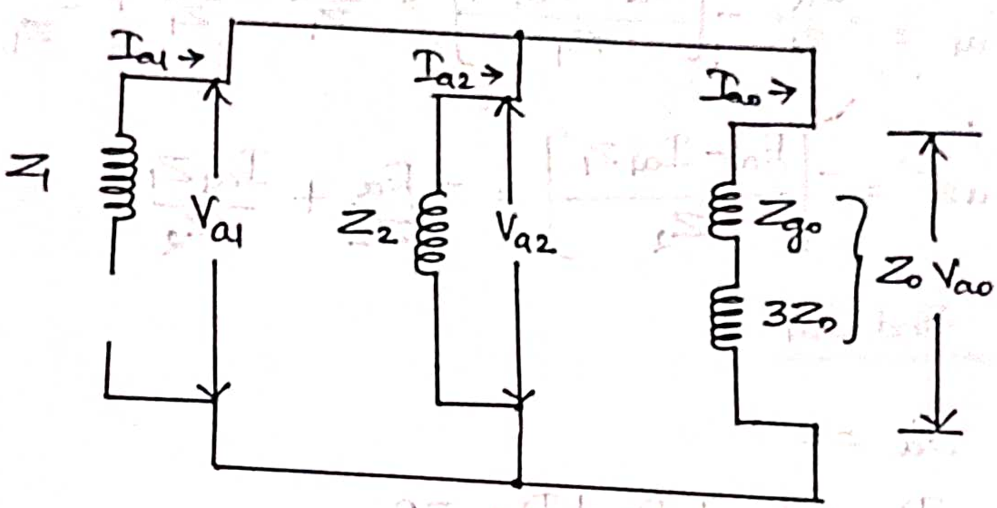
$$I_{a1}\left[1 + Z_1\left(\frac{Z_0 Z_2}{Z_0 + Z_2}\right)\right] = E_a\left[\frac{Z_0 + Z_2}{Z_0 Z_2}\right]$$

on xlying throughout by $\frac{Z_0 Z_2}{Z_0+Z_2}$ we get,

$$I_{a1}\left[\frac{Z_0 Z_2}{Z_0+Z_2} + Z_1\right] = E_a$$

$$I_{a1} = \frac{E_a}{Z_1 + \frac{Z_0 Z_2}{Z_0+Z_2}}$$

Step : 5 Equivalent circuit :



Step : 6 Fault Current :

$$I_f = I_b + I_c$$

WKT, $I_b = I_{a0} + a^2 I_{a1} + a I_{a2}$

$I_c = I_{a0} + a I_{a1} + a^2 I_{a2}$

ie, $I_f = I_{a0} + a^2 I_{a1} + a I_{a2} + I_{a0} + a I_{a1} + a^2 I_{a2}$

$= 2I_{a0} + I_{a1}(a^2 + a) + I_{a2}(a + a^2)$

$$\boxed{I_f = 2I_{a0} + (a^2+a)(I_{a1} + I_{a2})}$$

1. A salient pole generator without dampers is rated 20MVA, 13.8KV and has a direct axes subtransient reactance of 0.25 p.u. The negative and zero sequence reactances are 0.35 and 0.1 p.u respectively. The neutral of the generator is solidly grounded. Determine the subtransient current in the generator and the line to line voltages for subtransient conditions, when a single line to ground fault occurs at the generator terminals with generator operating unloaded at rated voltage. Neglect resistance.

Given,

$$Z_1 = 0.25j \text{ p.u}$$

$$Z_2 = 0.35j \text{ p.u}$$

$$Z_0 = 0.1j \text{ p.u}$$

For single line to ground fault,

$$I_{a0} = I_{a1} = I_{a2} = I_a/3$$

$$I_f = I_a = 3I_{a1}.$$

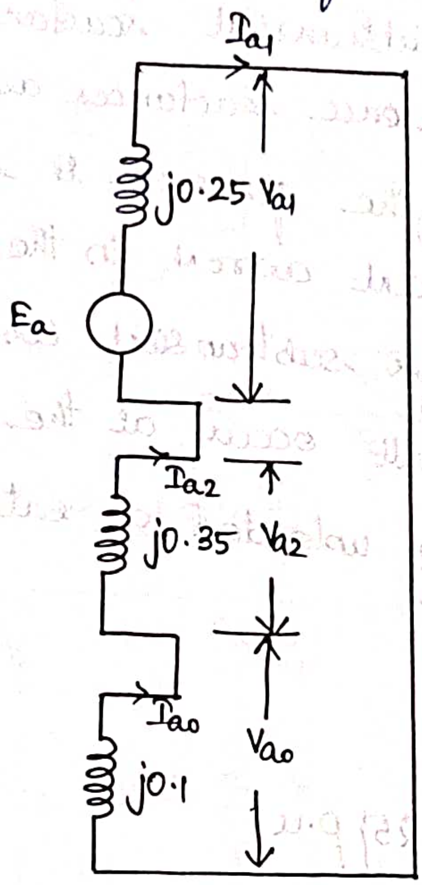
$$I_{a1} = \frac{E_a}{Z_0 + Z_1 + Z_2}$$

Prefault voltage $E_a = V^\circ = 1\angle 0^\circ$

$$I_{a1} = \frac{1\angle 0^\circ}{0.25j + 0.35j + 0.1j}$$

$$= -1.428j \text{ p.u}$$

The equivalent ckt is as follows. (connection of sequence n/ws)



Fault current I_f :- [Subtransient Current]

$$I_f = 3I_{a1}$$

$$= 3\times(-j1.4286)$$

$$= -j4.285 \text{ p.u.}$$

$$= 4.285\angle-90^\circ \text{ p.u.}$$

Actual Current :

$$= \text{p.u Value} \times \text{Base Current} (I_b).$$

$$= 4.285\angle-90^\circ \times \left[\frac{KVA_b}{\sqrt{3}\times KV_b}\right]$$

$$= 4.285\times\angle-90^\circ\times\left[\frac{2\times10^3}{\sqrt{3}\times 13.}\right]$$

$KV_b = 13.$

$MVA_b = 2$

$KVA_b = 2\times10^3$

$= 4.2857j \times 836.7$

$= -3.585 j$

$= 3.585 \angle -90^\circ$ KA.

$= 3.585$ KA.

Line to Line Voltages:

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}$$

$V_a = V_{a0} + V_{a1} + V_{a2}$.

$V_b = V_{a0} + a^2 V_{a1} + a V_{a2}$

$V_c = V_{a0} + a V_{a1} + a^2 V_{a2}$.

V_{a0}, V_{a1}, V_{a2} ?

$V_{a0} = -Z_0 I_{a0}$

$= -j0.1 \times -j1.4286$

$= -0.1428$ p.u.

$V_{a1} = E_a - Z_1 I_{a1}$.

$= 1\angle 0^\circ - (0.25j \times -j1.4286)$.

$= 0.6428$ p.u.

$V_{a2} = -Z_2 I_{a2}$

$= -(j0.35 \times -j1.4286)$

$= -0.50001$ p.u.

$V_a = V_{a0} + V_{a1} + V_{a2}$

$= -0.1428 + 0.6428 + (-0.5)$

$= 0.$

$a^2 = 1\angle 240°$

$= -0.5 - 0.866j$

$a = 1\angle 120°.$

$= -0.5 + j0.866.$

$V_b = V_{a0} + a^2 V_{a1} + aV_{a2}.$

$= -0.1428 + (-0.5 - 0.866j)(0.6428)$
$+ (-0.5 + j0.866)(-0.5)$

$= -0.215 - j0.9866$ p.u.

$V_c = V_{a0} + aV_{a1} + a^2 V_{a2}.$

$= -0.1428 + (-0.5 + j0.866)(0.6428) + (-0.5 - 0.866j)(-0.5)$

$= -0.215 + j0.9866$ p.u

line to line Voltages,

$V_{ab} = V_a - V_b = 0 - V_b = 0.215 + j0.9866$ p.u.
$= 1.01\angle 77.7$

$V_{bc} = V_b - V_c = -j1.9732 = 1.98\angle 270°$

$V_{ca} = V_c - V_a = V_c - 0 = -0.215 + j0.9866$ p.u
$= 1.01\angle 102.3°$

Actual Value of Voltage:

Actual Value = P.u x phase voltage.

Phase Voltage $= V_L/\sqrt{3} = 13.8/\sqrt{3} = 7.96$ KV.

$V_{ab} = 1.01\angle 77.7 \times 7.96$ KV. $= 8.05\angle 77.7$ KV.

$$V_{bc} = 1.98\angle 270^\circ \times 7.97$$

$$= 15.78\angle 270^\circ \text{ kV.}$$

$$V_{ca} = 1.01\angle 102.3^\circ \times 7.97$$

$$= 8.05\angle 102.3^\circ \text{ kV.}$$

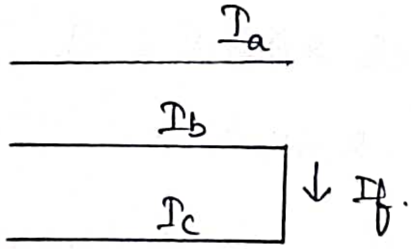
2. For the subtransient current & line-line Voltages at the fault under subtransient conditions when a line to line fault occurs b/w phase b and c at the terminals of the generator described in previous prob. Assume that the generator is unloaded & operating at rated terminal Voltage when the fault occurs. Neglect resistance.

Given:

$$Z_1 = j0.25 \text{ p.u}$$

$$Z_2 = j0.35 \text{ p.u}$$

$$Z_0 = j0.1 \text{ p.u}$$



$I_a = 0$; $I_b + I_c = 0$; $I_b = -I_c$; $I_f = I_b = -I_c$.

$V_{a1} = V_{a2}$; $I_{a0} = 0$; $I_{a1} = -I_{a2}$; $I_{a1} = \dfrac{E_a}{Z_1 + Z_2}$; $V_{a0} = 0$.

$$V_{a1} = E_a - Z_1 I_{a1}$$

$$I_{a1} = \frac{E_a}{Z_1+Z_2}$$

$$= \frac{1\angle 0^\circ}{j0.25+j0.35}$$

$$= \frac{1}{j0.6}$$

$$= -1.667j$$

$$I_{a1} = 1.667\angle -90^\circ \text{ p.u}$$

KKT,

$$I_{a2} = -I_{a1}$$

$$I_{a2} = 1.667j \text{ (or) } 1.667\angle 90^\circ \text{ p.u.}$$

$$I_{a0} = 0.$$

<u>Line currents</u> :-

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$

$$I_a = I_{a0} + I_{a1} + I_{a2}$$

$$I_b = I_{a0} + a^2 I_{a1} + a I_{a2}$$

$$I_c = I_{a0} + a I_{a1} + a^2 I_{a2}$$

$$a = -0.5 + j0.866$$

$$a^2 = -0.5 - j0.866.$$

$$I_a = 0 + (-1.667j) + (1.667j) = 0.$$

$$I_b = I_{a0} + a^2 I_{a1} + a I_{a2}.$$

$$= 0 + (-0.5 - j0.866)(-j1.667) + (-0.5 + j0.866)(1.667j)$$

$$= -2.886 \text{ p.u}$$

$$I_c = I_{a0} + a I_{a1} + a^2 I_{a2}$$

$$= 0 + (-0.5 + j0.866)(-1.667j) + (-0.5 - j0.866)(1.667j)$$

$$= 2.886 \text{ p.u}$$

Actual Current Values :

$$I_b = \frac{KVA_b}{\sqrt{3} \times KV_b} = \frac{20 \times 10^3}{\sqrt{3} \times 13.8} = 836.7 A.$$

Actual current = p.u Value × Base current.

$$I_a = 0$$

$$I_b = -2.886 \times 836.7$$

$$= -2416 \text{ A} = 2416 \angle 180^\circ.$$

$$I_c = 2.886 \times 836.7$$

$$= 2416 \text{ A}.$$

Fault Current I_f :

$$I_f = |I_b|$$

$$= 2416 \text{ A}.$$

Line to Line Voltages:-

$$V_{ab} = V_a - V_b$$

$$V_{bc} = V_b - V_c$$

$$V_{ca} = V_c - V_a$$

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}$$

$V_{a0} = 0$

$$V_{a1} = E_a - Z_1 I_{a1}$$

$$= 1 - (-1.667j)(j0.25)$$

$V_{a1} = 0.583$ p.u.

WKT, $V_{a1} = V_{a2}$

ie, $V_{a2} = 0.583$ p.u

From above said eqn.,

$$V_a = V_{a0} + V_{a1} + V_{a2}.$$

$$= 0 + 0.583 + 0.583$$

$$= 1.166 \text{ p.u}$$

$$V_b = V_{a0} + a^2 V_{a1} + a V_{a2}$$

$$= 0 + (-0.5 - j0.866)(0.583) + (-0.5 + j0.866)(0.583)$$

$V_b = -0.583$ p.u

$V_b = V_c$

ie $V_c = -0.583$ p.u

Line Voltages are,

$V_{ab} = V_a - V_b = 1.166 - (-0.583) = 1.749$ p.u.

$V_{bc} = V_b - V_c = -0.583 + 0.583 = 0$ p.u.

$V_{ca} = V_c - V_a = -0.583 - 1.166 = -1.749$ (or) $1.749\angle 180^\circ$ p.u

Actual line Voltages.

= P.u Value × phase Voltage.

$V_{ab} = 1.749 \times (13.8/\sqrt{3})$ kV

$\frac{13.8\text{kV}}{\sqrt{3}} = 7.967$ kV.

$= 13.935$ kV.

$V_{bc} = 0$

$V_{ca} = -1.749 \times 7.967$ kV.

$= -13.935$

$= 13.935\angle 180^\circ$ kV.

---

3. Double-line to ground fault for previous problem.

For DLG fault,

$I_{a1} = -(I_{a2} + I_{a0})$

$V_{a0} = V_{a1} = V_{a2}$

$V_b = V_c = 0.$

$$I_a = \frac{E_a}{Z_1 + \frac{Z_2 Z_0}{Z_2 + Z_0}}$$

$$= \frac{1 + j0}{j0.25 + (0.35j \times j0.1)/(j0.35 + j0.10)}$$

$$= \frac{1.0}{j0.25 + j0.0778}$$

$$= \frac{1.0}{j0.3278}$$

$$= \underline{\underline{-j3.05}}$$

$$V_{a1} = V_{a0} = V_{a2} = E_a - I_{a1} Z_1$$

$$= 1 - (-j3.05)(j0.25)$$

$$= 1.0 - 0.763$$

$$= 0.237 \text{ p.u}$$

$$I_{a2} = \frac{-V_{a2}}{Z_2} = \frac{-0.237}{j0.35} = \underline{\underline{j0.68 \text{ p.u}}}.$$

$$I_{a0} = \frac{-V_{a0}}{Z_0} = \frac{-0.237}{j0.10} = \underline{\underline{j2.37 \text{ p.u}}}$$

WKT,

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$

ie, $I_a = I_{a0} + I_{a1} + I_{a2}$

$= j2.37 - j3.05 + 0.68j$

$I_a = 0$

$I_b = I_{a0} + a^2 I_{a1} + a I_{a2}$

$= j2.37 + (-0.5 - j0.866)(-j3.05) + (-0.5 + j0.866)(j0.68)$

$= j1.525 - 2.641 - j0.34 - 0.589 + j2.37$

$= -3.230 + j3.555$

$= 4.80 \angle 132.3^\circ$ p.u

$I_c = I_{a0} + a I_{a1} + a^2 I_{a2}$

$= j2.37 + (-0.5 + j0.866)(-j3.05) + (-0.5 - j0.866)(j0.68)$

$= j1.525 + 2.641 - j0.34 + 0.589 + j2.37$

$= 3.230 + j3.555$

$= 4.80 \angle 47.7^\circ$ p.u

Fault Current : I_f

$I_f = I_b + I_c$

$= -3.230 + j3.555 + 3.230 + j3.555$

$= j7.11$ p.u

$V_a = V_{a0} + V_{a1} + V_{a2}$

$= 0.237 + 0.237 + 0.237$

$= 0.711$ p.u

$V_b = V_c = 0$.

$V_{ab} = V_a - V_b = 0.711$ p.u

... $1.2815 \angle 90$ A.

$V_{bc} = 0$

$V_{ca} = -0.711$ p.u.

Actual Values of Current:

$I_b = 836.7$ A.

$\therefore I_a = 0 \times 836.7 = 0$

$I_b = 4.8\angle 132.3^\circ \times 836.7 = 4017\angle 132.3^\circ$ A.

$I_c = 4.8\angle 47.7^\circ \times 836.7 = 5951\angle 90^\circ$ A.

$V_{ab} = V_{ab} \times$ phase Voltage.

$= 0.711 \times 7.9$ kV

$= 5.66\angle 0^\circ$ kV.

$V_{bc} = 0$

$V_{ca} = -0.711 \times 7.97$

$= 5.66\angle 180^\circ$ kV.

Univ. qn.

4. A generator of negligible resistance having 1 p.u Voltage behind transient reactance is subjected to different types of faults:

Type of fault	Resulting fault current in p.u.
3-ϕ	3.33
LL	2.23
LG	3.01

Calculate the per unit Value of 3 sequence reactance.

Case (i) 3-ϕ fault:

The reactance during 3-ϕ fault, ie symmetrical fault is positive sequence reactance. (X_1)

$$X_1 = \frac{E_a}{I_f}$$

$$= \frac{1}{3.33}$$

$$X_1 = 0.3 \text{ p.u}$$

Case (ii) LL fault:

Here. $X_0 = 0$.

$$I_{a1} = \frac{E_a}{jX_1 + jX_2}$$

$$|I_{a1}| = \frac{E_a}{X_1 + X_2}.$$

$$I_f = I_b = -I_c.$$

ie, $I_b = I_{a0} + a^2 I_{a1} + a I_{a2}$.

$I_{a0} = 0$.

$= 0 + a^2(I_{a1}) + a I_a$. $I_{a1} = -I_{a2}$.

$= I_{a1}(a^2 - a)$

$= (-0.5 - j0.866 + 0.5 - j0.866) I_{a1}$

$$I_f = -j1.732 I_{a1}$$

given: $I_f = 2.23$ p.u.

$$I_{a1} = \frac{2.23}{-j1.732}$$

$$I_{a1} = 1.2875j$$

$$|I_{a1}| = 1.2875 \angle 90^\circ \text{ A}.$$

$$I_{a1} = \frac{E_a}{X_1 + X_2}$$

$$1.2875 = \frac{1}{0.3 + X_2}$$

$$X_2 = 0.48 \text{ p.u}$$

Case (iii) L-G fault:-

$$|I_{a1}| = \frac{E_a}{X_1 + X_2 + X_0}$$

$$I_f = 3.01 \text{ p.u.}$$

$$I_f = 3 I_{a1} = I_a$$

$$I_{a1} = \frac{3.01}{3}$$

$$I_{a1} = 1.0033$$

$$X_0 = \frac{E_a}{|I_{a1}|} - X_1 - X_2$$

$$= \frac{1}{1.0033} - 0.3 - 0.48$$

$$= 0.22 \text{ p.u.}$$

Positive sequence reactance, $X_1 = 0.3$ p.u.

Negative sequence reactance, $X_2 = 0.48$ p.u.

Zero sequence reactance, $X_0 = 0.22$ p.u.

Network Modelling by means of sequence Bus impedance Matrices :

The positive sequence bus impedance matrix Z^+ is computed from positive sequence n/w. Using bus building algorithm.

Ill'y negative and zero sequence bus impedances ($Z^- \& Z^0$) are calculated.

ie

$$Z^+ = \begin{bmatrix} Z_{11}^+ & Z_{12}^+ & Z_{13}^+ \\ Z_{21}^+ & Z_{22}^+ & Z_{23}^+ \\ Z_{31}^+ & Z_{32}^+ & Z_{33}^+ \end{bmatrix}$$

$$Z^- = \begin{bmatrix} Z_{11}^- & Z_{12}^- & Z_{13}^- \\ Z_{21}^- & Z_{22}^- & Z_{23}^- \\ Z_{31}^- & Z_{32}^- & Z_{33}^- \end{bmatrix}$$ for: 3 bus

$$Z^0 = \begin{bmatrix} Z_{11}^0 & Z_{12}^0 & Z_{13}^0 \\ Z_{21}^0 & Z_{22}^0 & Z_{23}^0 \\ Z_{31}^0 & Z_{32}^0 & Z_{33}^0 \end{bmatrix}$$

* For analysis 3 sequence impedances are assembled into Z_{bus} of dimensions $3n \times 3n$, where n = no. of buses.

ie: $n = 3$

ie $(3\times3) \times (3\times3)$

9×9 matrix.

$$Z_{bus} = \left[\begin{array}{ccc|ccc|ccc} Z_{11}^{+} & 0 & 0 & Z_{12}^{+} & 0 & 0 & Z_{13}^{+} & 0 & 0 \\ 0 & Z_{11}^{-} & 0 & 0 & Z_{12}^{-} & 0 & 0 & Z_{13}^{-} & 0 \\ 0 & 0 & Z_{11}^{0} & 0 & 0 & Z_{12}^{0} & 0 & 0 & Z_{13}^{0} \\ \hline Z_{21}^{+} & 0 & 0 & Z_{22}^{+} & 0 & 0 & Z_{23}^{+} & 0 & 0 \\ 0 & Z_{21}^{-} & 0 & 0 & Z_{22}^{-} & 0 & 0 & Z_{23}^{-} & 0 \\ 0 & 0 & Z_{21}^{0} & 0 & 0 & Z_{22}^{0} & 0 & 0 & Z_{23}^{0} \\ \hline Z_{31}^{+} & 0 & 0 & Z_{32}^{+} & 0 & 0 & Z_{33}^{+} & 0 & 0 \\ 0 & Z_{31}^{-} & 0 & 0 & Z_{32}^{-} & 0 & 0 & Z_{33}^{-} & 0 \\ 0 & 0 & Z_{31}^{0} & 0 & 0 & Z_{32}^{0} & 0 & 0 & Z_{33}^{0} \end{array}\right]$$

* Determine the prefault & post-fault bus voltages and currents using the formulas given below.

Prefault Voltages:

* since the fault occurs when the system is balanced, all prefault bus voltages contains only positive sequence components.

$$V_{pf} = \begin{bmatrix} V_1^{+} \\ 0 \\ 0 \\ \vdots \\ V_i^{+} \\ 0 \\ 0 \\ \vdots \\ V_n^{+} \end{bmatrix}$$

Post fault Voltages:

The post fault positive sequence bus Voltages are given by,

$$[V_f^+] = [V_{p.f}] + [Z^+][I_f]$$

Post fault Current:

$$I_f = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ -I_k^{f+} \\ \vdots \\ 0 \end{bmatrix}$$

Post fault positive sequence bus Voltages are,

$$V_1^{f+} = V_{p.f} - Z_{1K}^+ I_K^{f+}$$

$$\vdots$$

$$V_K^{f+} = V_{p.f} - Z_{KK}^+ I_K^{f+}.$$

$$\vdots$$

$$V_n^{f+} = V_{p.f} - Z_{nK}^+ I_K^{f+}.$$

* The prefault negative & zero sequence bus Voltages are zero.

the post fault negative sequence Voltages are,

$$V_1^{f-} = -Z_{1K}^- I_K^{f-}.$$

$$\vdots$$

$$V_K^{f-} = -Z_{KK}^- I_K^{f-}.$$

$$\vdots$$

$$V_n^{f-} = -Z_{nK}^- I_K^{f-}$$

* The post fault zero sequences bus Voltages are,

$$V_1^{f0} = - Z_{1K}^{0} I_K^{f0}.$$

$\vdots$

$$V_K^{f0} = - Z_{KK}^{0} I_K^{f0}.$$

$\vdots$

$$V_n^{f0} = - Z_{nK}^{0} I_K^{f0}.$$

Post fault line currents:

positive sequence line currents, $I_{ij}^{f+} = \dfrac{V_i^{f+} - V_j^{f+}}{Z_{ij}^{+}}.$

Negative sequence line current, $I_{ij}^{f-} = \dfrac{V_i^{f-} - V_j^{f-}}{Z_{ij}^{-}}$

Zero sequence line current, $I_{ij}^{f0} = \dfrac{V_i^{f0} - V_j^{f0}}{Z_{ij}^{0}}.$

Phase Voltages:

$$[V_p] = [T][V_s]$$

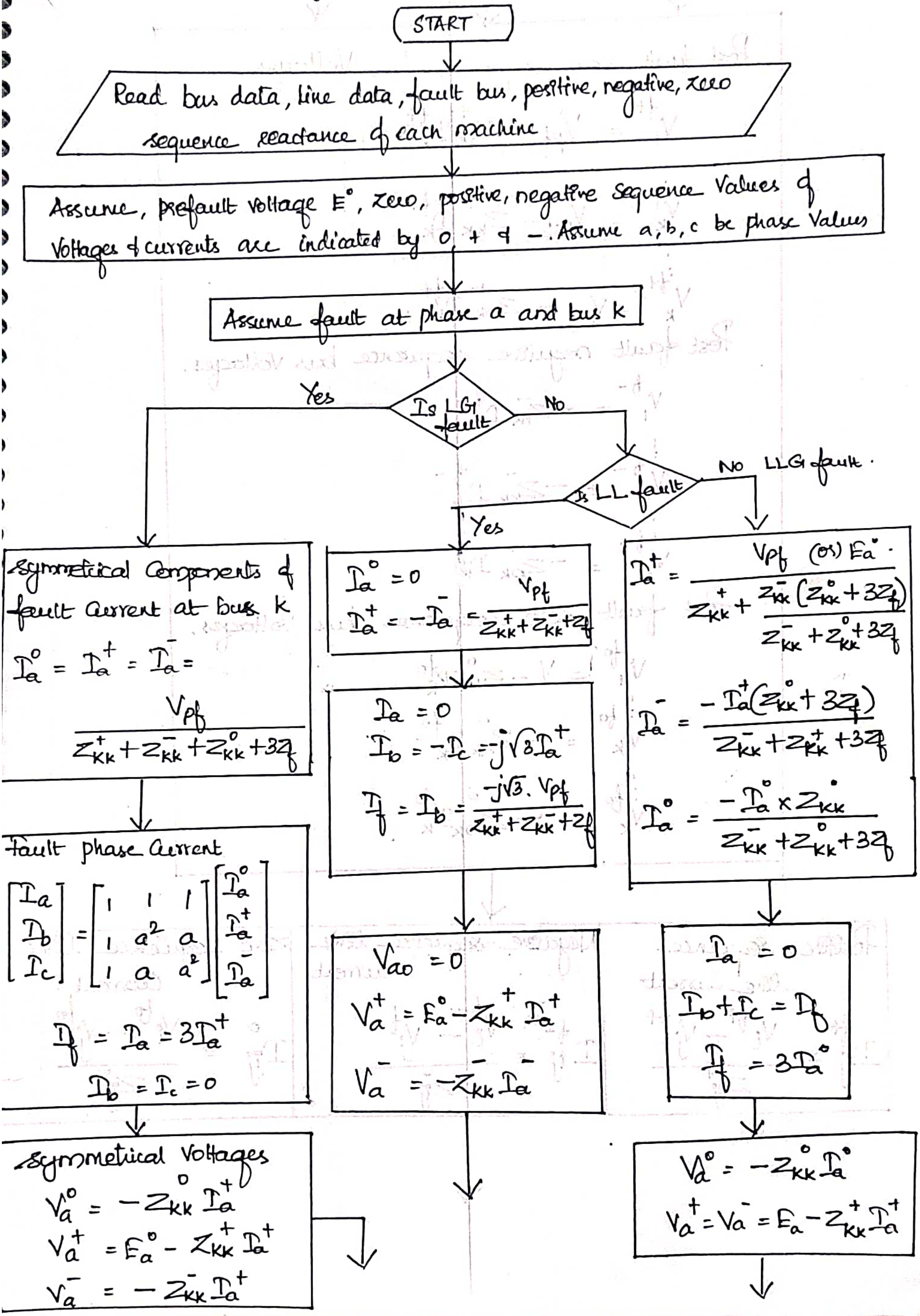
$$[I_p] = [T][I_s]$$

$[V_p],[I_p]$ = phase Voltage & currents.

$[V_s],[I_s]$ = Sequence Voltage & currents.

$$[T] = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$$

Flowchart for Unsymmetrical Fault Analysis :-



Post fault Zero sequence bus Voltages

$$V_1^{f+} = V_{pf} - Z_{1k}^{+} I_k^{f+}$$

$$\vdots$$

$$V_k^{f+} = V_{p.f} - Z_{kk}^{+} I_k^{f+}$$

$$\vdots$$

$$V_k^{f+} = V_{p.f} - Z_{nk}^{+} I_k^{f+}.$$

Post fault negative sequence bus Voltages.

$$V_1^{f-} = -Z_{1k}^{-} I_k^{f-}$$

$$\vdots$$

$$V_k^{f-} = -Z_{kk}^{-} I_k^{f-}$$

$$\vdots$$

$$V_n^{f-} = -Z_{nk}^{-} I_k^{f-}$$

Post fault Zero sequence bus Voltages.

$$V_1^{fo} = -Z_{1k}^{0} I_k^{fo}$$

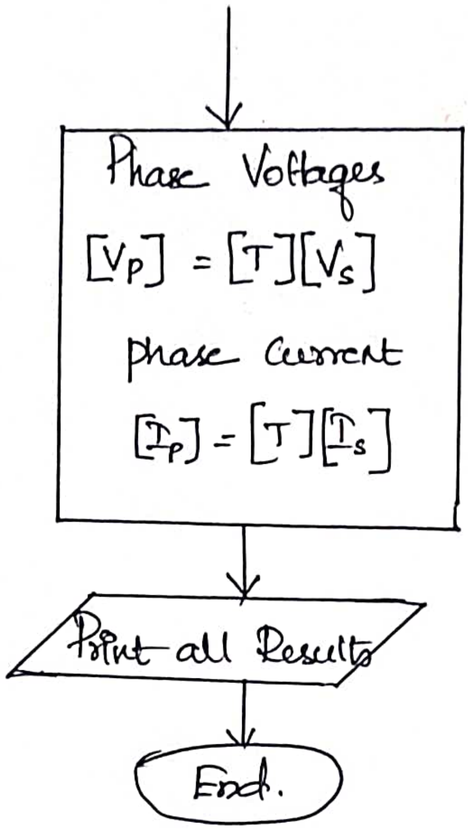
$$\vdots$$

$$V_k^{fo} = -Z_{kk}^{0} I_k^{fo}$$

$$\vdots$$

$$V_k^{fo} = -Z_{nk}^{0} I_k^{fo}.$$

Positive sequence line current	Negative sequence line current	Zero sequence line Current.
$I_{ij}^{f+} = \dfrac{V_i^{f+} - V_j^{f+}}{Z_{ij}^{+}}$	$I_{ij}^{f-} = \dfrac{V_i^{f-} - V_j^{f-}}{Z_{ij}^{-}}$	$I_{ij}^{0} = \dfrac{V_i^{fo} - V_j^{fo}}{Z_{ij}^{0}}$


Phase Voltages
[Vp] = [T][Vs]
phase current
[Ip] = [T][Is]
Print all Results
End.

5: STABILITY ANALYSIS

Stability:

It is the ability of power system to remain in a state of equilibrium under normal operating conditions and to return to stable or normal operation after have been subjected to some form of disturbance.

Stability Problems:

* Short circuit on transmission line.
* Sudden Loss of generation.
* Sudden loss of load.
* Sudden Loss of excitation in a generator.
* Sudden Loss of tie line b/w 2 subsystem.
* Flashover of insulators.
* Maloperation of protective devices.
* closing /opening of Circuit Breaker by operators. etc.

Importance of Stability Analysis

* Stability studies gives the information that the system can withstand the transient conditions like high magnitude of Voltage, frequency etc..
* stability studies are needed when new generating station and transmission facilities are planned.

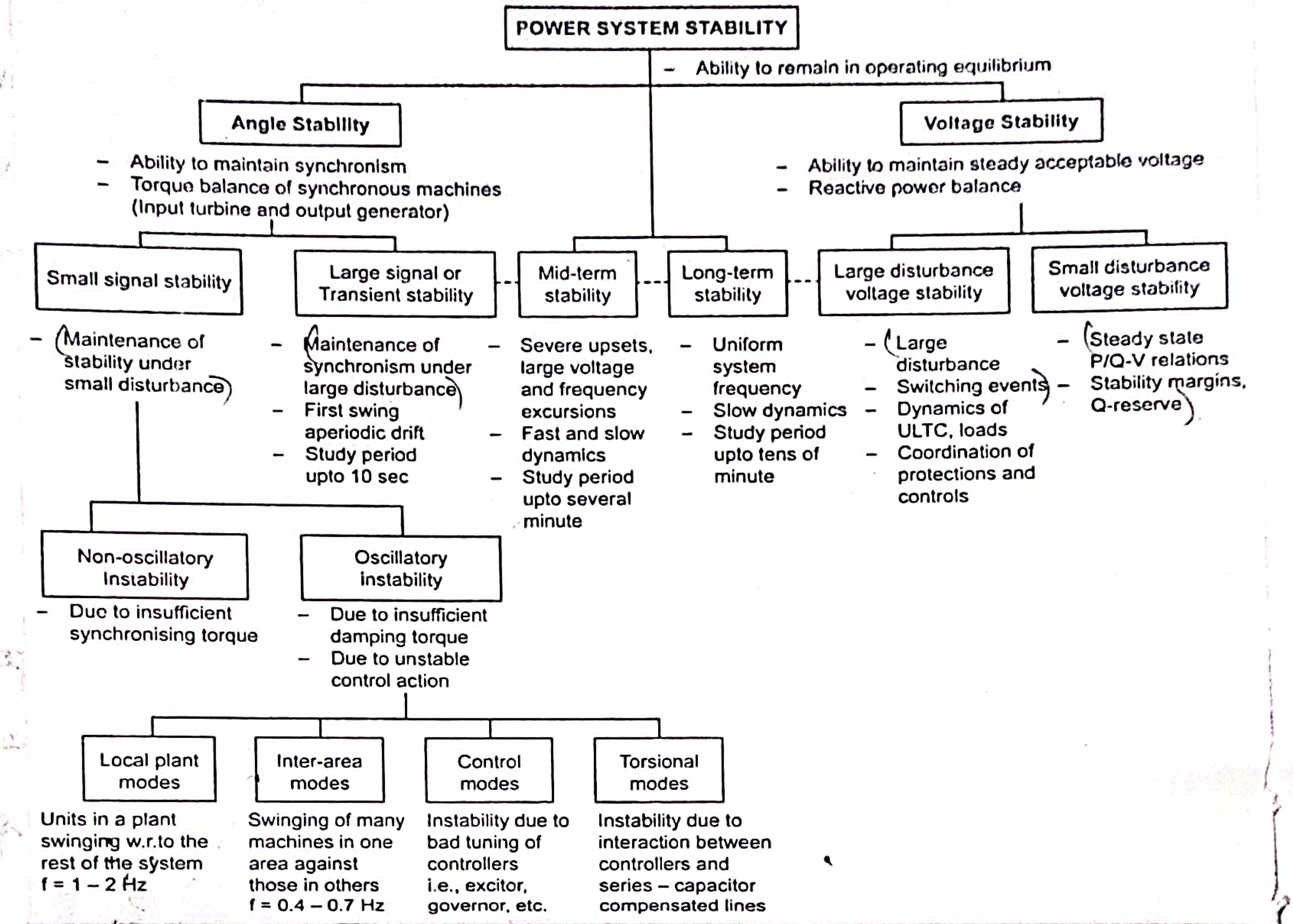
* Stability studies are useful in determining the nature of relaying system needed, critical clearing time of circuit Breakers i.e. design of protection equipments.
* Stability studies are more helpful in determining power transfer capability between different systems.

Adverse Effects of Instability:

* It has bad effect on service to consumer loads.
* A generator has no longer constitutes a reliable source of electric power at proper speed when one machine falls out of step with others in the system.
* When a large synchronous machine falls out of step it has a disturbing effect on voltage over a wide range which is highly injurious to other equipments.
* A condenser no longer maintains proper voltage at its terminal.
* The resynchronization procedure is quite lengthy and hence there will be heavy economic loss to the power plant industries.
* Instability means loss of synchronism of synchronous machines in the power systems.
* For analysis the multimachine system can be converted [illegible] equivalent single machine system connected to [illegible] Practical Power S/m has no. of machines running i

synchronism. It is called the Single Machine Infinite Bus. (SMIB)

Classification of Power System Stability :-



Rotor Angle Stability :-

Rotor angle stability is the ability of interconnected synchronous machines of a power system to remain in synchronism.

The stability problem involves the study of electromechanical oscillations involving exchange of energy between network and generator-mechanical system. at or close to power frequency.

The problem is the manner in which the output power of synchronous machines vary as rotor oscillates.

If 2 or more synchronous machines are connected to a common bus, the frequency of stator voltages, current and rotor mechanical speed of each machine is should be synchronised.

Also practically there is a angle difference b/w revolving field and rotor field which depends on the electrical torque in synchronous machines, that is the output of generator.

i) Small signal (or) Small disturbance stability.

ii) Transient stability (or) Large signal stability.

Small signal stability:

Small signal stability is the ability of the power system to maintain synchronism under small disturbances.

Such disturbances occur continually on the system because of small variations in loads & generation.

Instability may result due to following 2 forms.

i) Steady increase in rotor angle due to lack of sufficient synchronising torque.

ii) Rotor oscillations of increasing amplitude due to lack of sufficient damping torque.

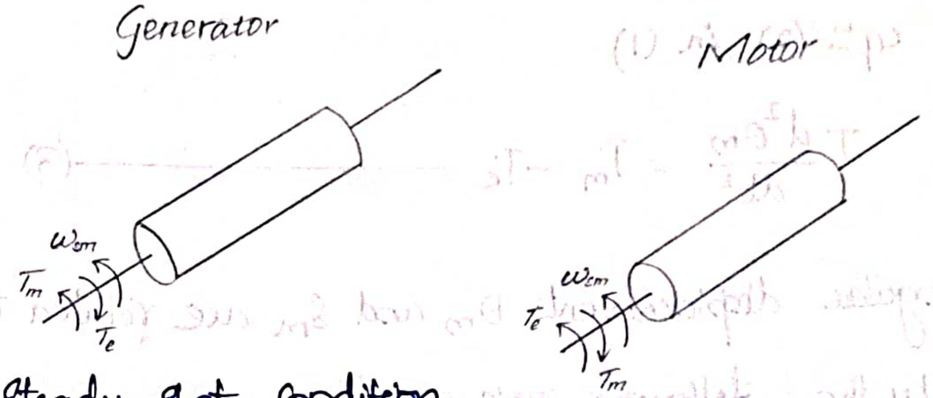
The nature of system response to small disturbance depends on a no. of factors including the initial operating, transmission strengths, the type of generator excitation

Swing Equation: 5C

* The rotor of a synchronous machine is subjected to two torques, T_e and T_m.

T_e : Net electrical torque (N-m)

T_m : Net mechanical torque (N-m)



* Under steady state condition,

$$T_a = T_e$$

ie $$T_a = T_m - T_e = 0.$$

There will be no acceleration or decceleration torque, & the machine runs at synchronous speed.

1 * Under disturbance, there will be a disturbance between two torques. Then accelerating $(T_m > T_e)$ or decceleating torque $(T_m <$

will given by,

$$T_a = T_m - T_e \text{ ——— } (1)$$

2 * By Newton's Second Law,

$$T_a \propto \frac{d^2\theta_m}{dt^2}$$

ie. accelerating torque is proportional to angular acceleration

$$T_a = J\frac{d^2\theta_m}{dt^2} \quad \text{———— (2)}$$

θ_m : Angular displacement of rotor with respect to stationary reference axis in mech.rad

J : moment of inertia ($kg\text{-}m^2$)

Subs eqⁿ. (2) in (1)

$$J\frac{d^2\theta_m}{dt^2} = T_m - T_e \quad \text{———— (3)}$$

step:3 The angular displacements θ_m and δ_m are related to synch speed by the following eqⁿ.

$$\theta_m = \omega_{sm}t + \delta_m \quad \text{———— (4)}$$

δ_m : Angular displacement of rotor with respect to synchronously rotating reference frame in mech.rad

ω_{sm} : Synchronous angular speed of rotor in mech.rad

differentiating eqⁿ. no. (4) w.r. to t.

$$\frac{d\theta_m}{dt} = \omega_{sm} + \frac{d\delta_m}{dt}$$

Again differentiating the above eqⁿ.

$$\frac{d^2\theta_m}{dt^2} = \frac{d^2\delta_m}{dt^2} \quad \text{———— (5)}$$

Subs. eqⁿ. (5) in (3).

$$J \frac{d^2\delta_m}{dt^2} = T_m - T_e \quad \text{———— (6)}$$

WKT, $P = \omega T$

From above eqⁿ,

$$P_{m,act} = \omega_{sm} T_m$$

$$T_m = \frac{P_{m,act}}{\omega_{sm}} \quad \text{———— (7)}$$

& $$P_{e,act} = \omega_{sm} T_e$$

$$T_e = \frac{P_{e,act}}{\omega_{sm}} \quad \text{———— (8)}$$

ω_{sm} : synchronous angular speed of the rotor (mech. rad/sec)

Subs. (7) & (8) in eqⁿ. (6)

$$J \frac{d^2\delta_m}{dt^2} = \frac{P_{m,act}}{\omega_{sm}} - \frac{P_{e,act}}{\omega_{sm}}$$

$$J \frac{d^2\delta_m}{dt^2} = \frac{P_{m,act} - P_{e,act}}{\omega_{sm}}$$

$$J\omega_{sm} \frac{d^2\delta_m}{dt^2} = P_{m,act} - P_{e,act} \quad \text{———— (9)}$$

Inertia Constant, H

H is defined as the ratio of stored kinetic energy in MJ to the machine rating MVA.

$$H = \frac{\text{Stored Kinetic Energy (MJ/MVA)}}{\text{Power rating (MVA)}}$$

$$H = \frac{\frac{1}{2} J \omega_{sm}^2}{S}$$

$$HS = \frac{1}{2} J\omega_{sm} \cdot \omega_{sm}$$

$$J\omega_{sm} = \frac{2HS}{\omega_{sm}} \quad \text{———— (10)}$$

Subs. eq. (10) in eq. (9)

$$\frac{2HS}{\omega_{sm}} \frac{d^2\delta_{sm}}{dt^2} = P_{m,act} - P_{e,act} \quad \text{———— (11)}$$

step : 6 The mechanical and electrical angular speeds are related to the number of poles in synchronous machines & is g. by

$$\omega_{sm} = \frac{2}{P}\omega_s \quad \text{———— (12)}$$

$$\delta_{sm} = \frac{2}{P}\delta \quad \text{———— (13)}$$

P = No. of poles in synchronous machine.

Subs (12) & (13) in (10).

$$\frac{2HS}{(2/P)\omega_s} \frac{d^2(2/P\,\delta)}{dt^2} = P_{m,act} - P_{e,act}$$

$$\frac{2HS}{\omega_s} \frac{d^2\delta}{dt^2} = P_{m,act} - P_{e,act} \quad \text{———— (13)}$$

step : 7 Subs → $\omega_s = 2\pi f$ in eq. (13).

$$\frac{2HS}{2\pi f} \frac{d^2\delta}{dt^2} = P_{m,act} - P_{e,act}$$

$$\frac{HS}{\pi f} \frac{d^2\delta}{dt^2} = P_{m,act} - P_{e,act}.$$

P.u Value of mechanical Power, $P_m = \frac{P_{m,act}}{S}$

P.u Value of electrical Power, $P_e = \frac{P_{e,act}}{S}$

From Eq:- (14)

$$\frac{H}{\pi f}\frac{d^2\delta}{dt^2} = \frac{P_{m,act}}{S} - \frac{P_{e,act}}{S}$$

$$\boxed{\frac{H}{\pi f}\frac{d^2\delta}{dt^2} = P_m - P_e} \quad \text{——— (15)}$$

Eq:-(15) is called Swing Equation.

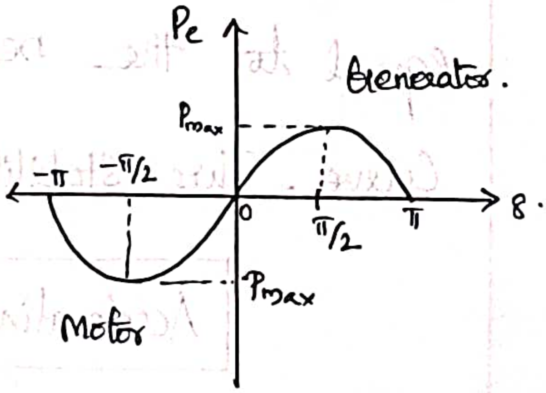
* It is the fundamental equation which governs the dynamics of the synchronous machine rotor.

Power Angle Equation:

The equation relating the electrical power generated (P_e) to the angular displacement of the rotor (δ) is called power angle equation.

$$P_e = P_{max} \sin\delta$$

$$P_{max} = \frac{|E_1||E_2|}{X_{12}}$$



E_1 - Generator voltage. (bus-1)

E_2 - Voltage at receiving end. (bus-2)

X_{12} - Transfer reactance b/w (bus 1 & 2).

Transient Stability:

The transient stability of a system is concerned with the study of system behaviour for large disturbances

Eg. Short circuits.

Switching heavy loads.

Equal Area Criterion:

The transient stability analysis of simple system can be performed by using a simple criterion called Equal area criterion.

Statement of Equal area criterion:

The system is stable if the area under $P_a - \delta$ curve reduces to zero at some value of δ. This is possible only if the positive area (accelerating area) under $P_a - \delta$ curve is equal to the negative (decelerating) area under $P_a - \delta$ curve. This stability criterion is called equal area criterion

Accelerating area = Decelerating area.

Stability Criterion :-

(i) Stable System : $\frac{d\delta}{dt} = 0$.

(ii) Unstable System : $\frac{d\delta}{dt} > 0$

Consider the swing equation,

$$\frac{H}{\pi f}\frac{d^2\delta}{dt^2} = P_m - P_e \text{ ——— } (1)$$

① $P_m - P_e = P_a$
② $H/\pi f = M$
} ——— Subs in (1).

$$M\frac{d^2\delta}{dt^2} = P_a.$$

$$\frac{d^2\delta}{dt^2} = \frac{P_a}{M} \text{ ——— } (2)$$

Xly equation (2) by $\frac{2d\delta}{dt}$ on both sides,

$$2\frac{d\delta}{dt}\cdot\frac{d^2\delta}{dt^2} = \frac{2d\delta}{dt}\cdot\frac{P_a}{M}$$

WKT,

$$\frac{d}{dt}(x^2) = 2x\cdot\frac{dx}{dt}.$$

Similarly,

$$\frac{d}{dt}\left(\frac{d\delta}{dt}\right)^2 = 2\frac{d\delta}{dt}\cdot\frac{d^2\delta}{dt^2} \text{ ——— } (3).$$

Substitute eqⁿ (3) in above eqⁿ.

$$\frac{d}{dt}\left(\frac{d\delta}{dt}\right)^2 = \frac{2d\delta}{dt}\cdot\frac{P_a}{M}.$$

$$d\left(\frac{d\delta}{dt}\right)^2 = 2d\delta\cdot\frac{P_a}{M}.$$

Integrating both sides,

$$\int d\left(\frac{d\delta}{dt}\right)^2 = \int 2d\delta\cdot\frac{P_a}{M}.$$

$$\left(\frac{d\delta}{dt}\right)^2 = \frac{2}{M}\int_{\delta_0}^{\delta} P_a\cdot d\delta.$$

Taking square root on both sides,

$$\frac{d\delta}{dt} = \sqrt{\frac{2}{M}\int_{\delta_0}^{\delta} P_a\cdot d\delta}.$$

For stable system,

$$\frac{d\delta}{dt} = 0$$

$$\sqrt{\frac{2}{M}\int_{\delta_0}^{\delta} P_a . d\delta} = 0.$$

$$\int_{\delta_0}^{\delta} P_a . d\delta = 0. \quad \text{——— (4).}$$

* Integration means estimation of area under the curve.
* Hence the integral of Pa equal to zero refers to zero area.
* So it can be stated as the system is stable if area under $P_a - \delta$ curve reduces to zero at some value of δ.
* An area cannot be reduced to zero. But it can be made to zero if 2 area cancels each other.
* So if accelerating area is equal to decelerating area, the in $P_a - \delta$ curve, this condition is possible.

WKT,

$$P_a = P_m - P_e. \quad \text{——— (5).}$$

Substitute eqn: (5) in (4).

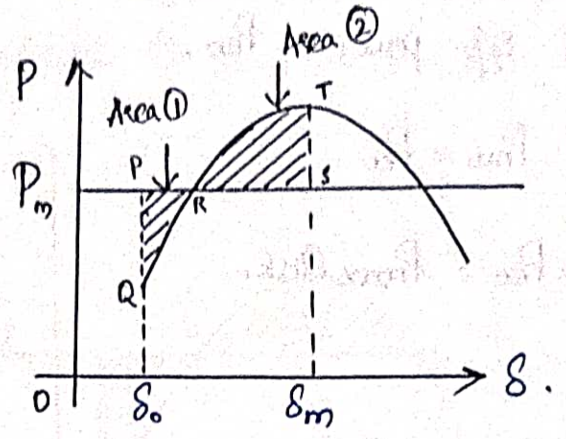
$$\int_{\delta_0}^{\delta} (P_m - P_e) . d\delta = 0.$$

$$\int_{\delta_0}^{\delta} P_m . d\delta - \int_{\delta_0}^{\delta} P_e . d\delta = 0.$$

(or)

$$\int_{\delta_0}^{\delta} P_m . d\delta = \int_{\delta_0}^{\delta} P_e . d\delta.$$

* For a given P-δ curve,



PQ represents $P_m - P_e$ is equal to accelerating power.

ST represents $P_e - P_m$ is equal to decelerating power.

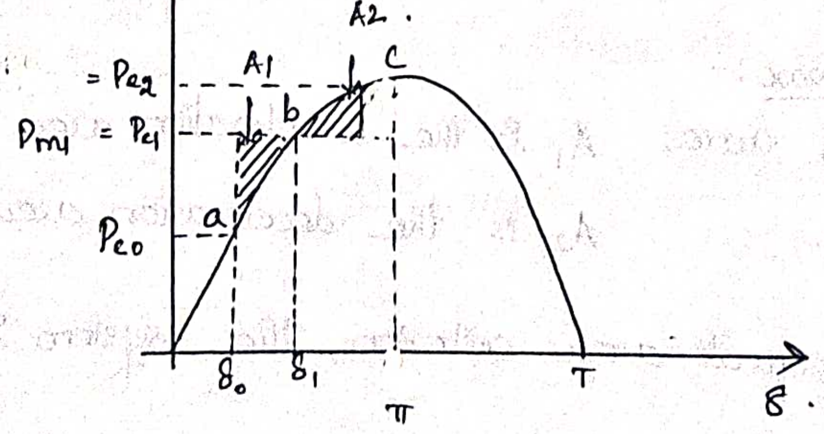
* If area 1 = area 2, then system will be stable.

<u>Transient Stability analysis for a sudden change in Mechanical i/p :-</u>

Consider a single generator feeding energy to infinite bus as shown in figure. The electrical power transmitted by the generator is given by,

$$P_e = \frac{|E_1||E_2|}{X} \sin\delta .$$

$$P_e = P_m \sin\delta .$$



steady state (P_{m0})

torque angle, δ_0.

mechanical input $p = P_{mo}$.

electrical o/p power = P_{eo}.

For ideal case, $P_{mo} = P_{eo}$

$$P_{mo} = P_{eo} = P_{max} \sin\delta.$$

Next level (P_{m1}).

* Mechanical i/p increased by prime mover to P_{m1}.
* Mechanical P > Electrical Power.
* Then this is accelerating Power (P_a)
* At this stage:

⇒ rotor speed increases / rotor angle increases (δ_1) / electrical Power increases, P_e (P_{e1})

⇒ then $P_{m1} = P_{e1}$.

Decelerating Power:-

* The inertia causes rotor angle to increase further (δ_2), Causes P_{e2} to increase. So $P_{e2} > P_m$. Hence its a decelerating power.

Derivation of δ_{max}:-

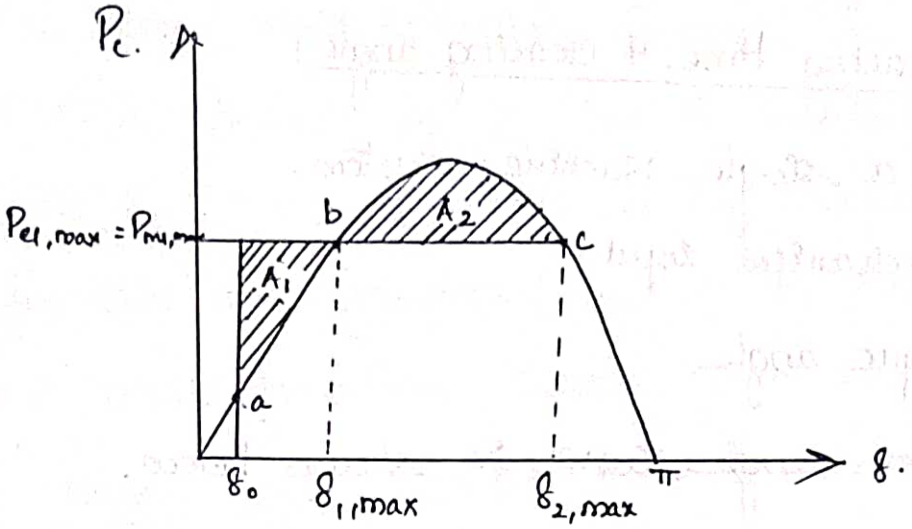
* In the P-δ curve A_1 is the acceleration area.

A_2 is the deceleration area.

* According to equal area criterion, the system is stable if,

$$\int_{\delta_0}^{\delta} P_a \cdot d\delta = 0.$$

* To satisfy the equal area criterion concept, A_1 should be made equal to A_2.



* This condition ($A_1 = A_2$) says that there is an upper limit for increase in mechanical power input, P_m.

* As the mechanical power is increased, a limiting condition is finally reached, where A_1 equals A_2.

* From the curve,

$$\delta_{1,max} = \pi - \delta_{2,max}$$

$$\therefore \quad \delta_{2,max} = \pi - \delta_{1,max} \quad \text{——— (6).}$$

WKT,

$$P_{m1,max} = P_{max} \sin \delta_{1,max} \quad \left[P_{m1,max} = P_{e1,max} \right] \text{ from curve.}$$

$$\sin \delta_{1,max} = \frac{P_{m1,max}}{P_{max}}$$

$$\delta_{1,max} = \sin^{-1}\left[\frac{P_{m1,max}}{P_{max}}\right] \quad \text{——— (7)}$$

Subs. (7) in (6).

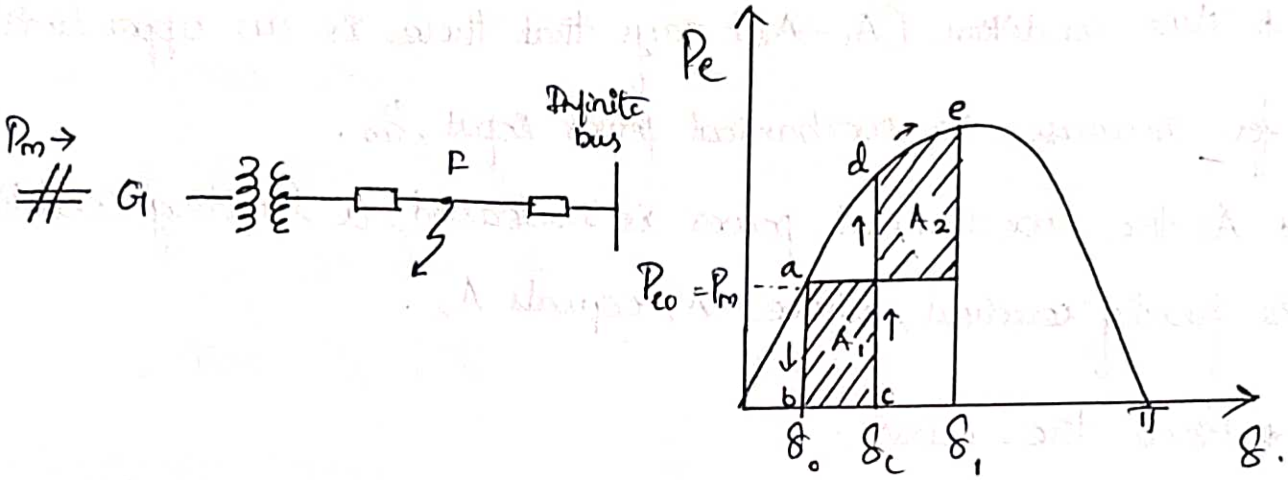
$$\delta_{2,max} = \pi - \sin^{-1}\left[\frac{P_{m1,max}}{P_{max}}\right]$$

* Further Increase in $P_{m,max}$ will make A_2 less than A_1.

So the system becomes unstable.

Clearing time & clearing angle:

* Consider a single Machine system.
* P_m : mechanical input.
* δ_0 : torque angle.
* The power-angle curve is shown below.



* The curve is smooth upto 'a'.
* A 3-ϕ fault occurs at point F. i.e. $P_e = 0$, so operating point drops to b.
* Now the operating point moves along bc.
* Let the fault be cleared at this point, δ_c and corresponding time be t_c.

δ_c : clearing angle.

t_c : Clearing time.

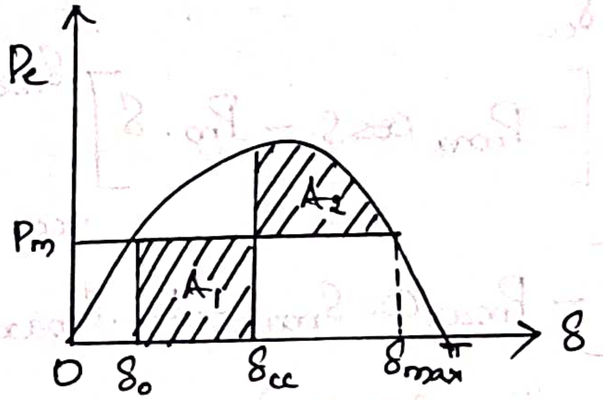
* Now fault is cleared by means of circuit Breaker, so the point shifts to 'd'. & keep moving to e.

* For the above said operation, $A_2 = A_1$ and the system is found to be stable.

* For the above said operation (abcde) the fault is assumed to be cleared at δ_c. But if the fault is not cleared at δ_c, then δ_1 continues to increase to δ_{max}.

* So for this new situation (fault is not cleared at δ_c) the fault would have been cleared at an angle δ_{cc}, called the critical clearing angle and the time corresponding to this angle is called critical clearing time, t_{cc}.

* Here also A_1 is found to be equal to A_2 for a given P_m as shown in figure.



* Again if fault is not cleared at δ_{cc}, then A_1 will not be equal to A_2, so the system becomes unstable.

Derivation (δ_{cc} & t_{cc})

Procedure:

Step:1 Find A_1 & A_2

Step:2 Equate ; $A_1 = A_2$

$$A_1 = \int_{\delta_0}^{\delta_{cc}} P_m \, d\delta.$$

(A_1 : acceleration area. at fault $P_e = 0$ so $P_a = P_m - 0$

$$= P_m \left[\delta \right]_{\delta_0}^{\delta_{cc}}.$$

$$A_1 = P_m \left[\delta_{cc} - \delta_0 \right] \quad \text{--- (1)}.$$

* After fault is cleared,

$$P_a = P_e - P_m.$$

$$= P_n$$

$$= P_{max} \sin\delta - P_m$$

* The deceleration Area A_2 :-

$$A_2 = \int_{\delta_{cc}}^{\delta_{max}} P_a \cdot d\delta.$$

$$= \int_{\delta_{cc}}^{\delta_{max}} [P_{max} \sin\delta - P_m] \cdot d\delta.$$

$$= \left[-P_{max} \cos\delta - P_m \cdot \delta \right]_{\delta_{cc}}^{\delta_{max}}.$$

$$= \left[-P_{max} \cos\delta_{max} - P_m \cdot \delta_{max} + P_{max} \cos\delta_{cc} + P_m \delta \right.$$

(Take P_{max}, P_m common) --- (2).

$$A_2 = P_{max} (\cos\delta_{cc} - \cos\delta_{max}) - P_m (\delta_{max} - \delta_{cc})$$

* For stable system $A_1 = A_2$.

so from eqn. (1) & (2). Equate $A_1 = A_2$.

$$\therefore P_m \left[\delta_{cc} - \delta_0 \right] = .$$

$$= -P_{max} \cos\delta_{max} - P_m \delta_{max} + P_{max} \cos\delta$$

$$+ P_m \delta_{cc}.$$

$$P_{max}\cos\delta_{cc} = P_m\delta_{cc} - P_m\delta_0 + P_{max}\cos\delta_{max} + P_m\delta_{max} - P_m\delta_{cc}$$

$$P_{max}\cos\delta_{cc} = P_m(\delta_{max} - \delta_0) + P_{max}\cos\delta_{max}.$$

$$\cos\delta_{cc} = \frac{P_m}{P_{max}}(\delta_{max} - \delta_0) + \frac{P_{max}}{P_{max}}\cos\delta_{max}.$$

$$\boxed{\delta_{cc} = \cos^{-1}\left[\frac{P_m}{P_{max}}(\delta_{max} - \delta_0) + \cos\delta_{max}\right].}$$

t_{cc} :-

Consider swing eqn.,

$$\frac{H}{\pi f}\frac{d^2\delta}{dt^2} = P_m - P_e.$$

⇒ During fault no electrical output will be there, $P_e = 0$.

$$\frac{H}{\pi f}\frac{d^2\delta}{dt^2} = P_m.$$

$$\frac{d^2\delta}{dt^2} = \frac{\pi f P_m}{H}$$

⇒ Integrate above equation,

$$\frac{d\delta}{dt} = \frac{\pi f P_m}{H}.t.$$

⇒ Integrating again, we get, $\int t = \frac{t^2}{2}$.

$$\delta = \frac{\pi f P_m}{H}.\frac{t^2}{2} + \delta_0$$

δ_0 = integral Constant

⇒ when $\delta = \delta_{cc}$; $t = t_{cc}$.

Subs. $\delta = \delta_{cc}$ & $t = t_{cc}$ in above eqⁿ.

$$\delta_{cc} = \frac{\pi f P_m}{2H} t_{cc}^2 + \delta_0 .$$

$$t_{cc}^2 = \frac{2H(\delta_{cc} - \delta_0)}{\pi f P_m}$$

$$\boxed{t_{cc} = \sqrt{\frac{2H(\delta_{cc} - \delta_0)}{\pi f P_m}}}$$

Methods of Improving Transient Stability:

* Transient stability can be improved by,

1. Reduction in the disturbing influence by minimizing fault severity & duration.
2. Increasing the restoring synchronizing forces.
3. Reduction of acceleration torque through control of prime mover
4. Reduction of accelerating torque by applying artificial load.

Traditional approaches to transient stability problem:-

1. Increase of system voltages by AVR.
2. Use of high speed excitation systems.

⇒ Increase in generator excitation during transient disturbance increases the internal voltage of the machine.

3. Reduction in system transfer reactance.

(i) Using transformer with low leakage reactance.

(ii) Series capacitor compensation of transmission lines.

4. Use of high speed reclosing Breakers.

5. Single pole operation of reclosing circuit Breakers.

6. Use of bundled Conductors.

7. High speed fault clearing.

8. Increasing no. of parallel lines between points.

9. Regulated Shunt Compensation.

10. Dynamic Breaking.

11. Single Pole Switching.

12. Generator tripping - selective tripping of generator units for severe transmission system contingency

Recent Trends :-

(1) HVDC links :

Using HVDC links employing thyristors reduces stability Problem. Because dc link is asynchronous. Hence different frequency system can be connected without losing stability.

Breaking Resistors :

Sometimes large loads can be suddenly lost. For that a resistive load called a breaking resistor is connected near the generator bus. This load compensates for some of the reduction of load on the generators.

iii) Bypass Valving:-

stability is improved by decreasing the mechanical input power to the turbine.

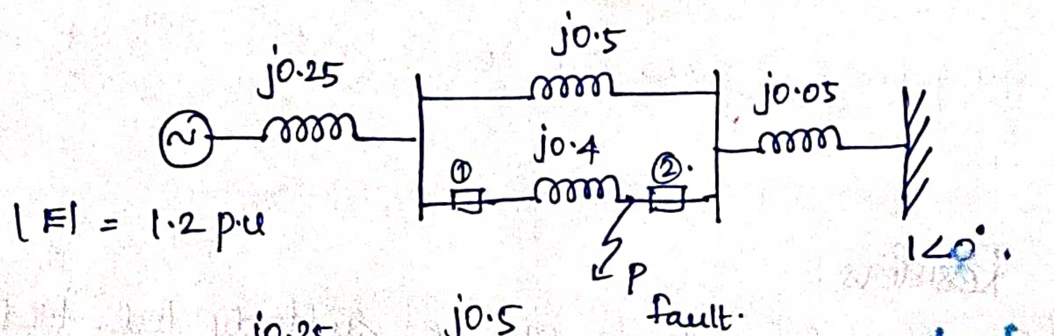
iv) Full load Rejection technique:

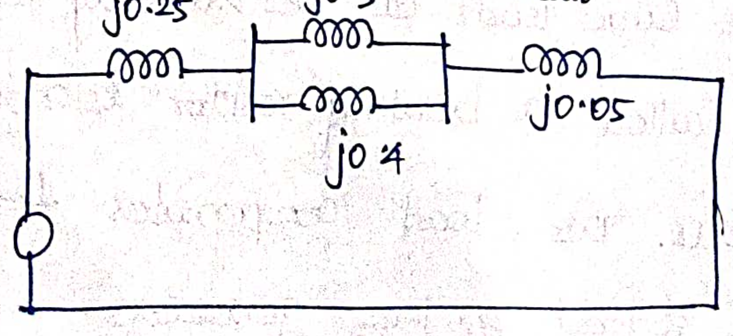
In some cases stability is difficult to maintain. Under this situation the full load rejection scheme is used, Here the unit is equipped with steam "bypass" system. After the fault system is recovered, that unit is resynchronised & connected again.

The disadvantage of this system is extra cost for bypass system.

Problem in Critical clearing time

1. A 3-φ fault is applied at point 'P'. Find the critical clearing angle for clearing the fault with simultaneous opening of the breakers 1 & 2. The reactance values of various components are indicated on the diagram. The generator is delivering 1.0 pu. power at the instant preceding the fault.





8.24.4 Runga Kutta Method

The R-K methods approximate the Taylor series solution ; however unlike the formal Taylor series solution, the R-K method do not require explicit evaluation of derivatives higher than the first. Depending on the number of terms effectively retained in the Taylor series, we have R-K methods of different orders.

The independent variable is given a set of predetermined value and the corresponding dependent variable are evaluated directly from a set of formulae. The method is self starting and does not require any successive approximations as used in the modified Euler's method.

8.24.5 Runga Kutta Fourth Order Approximation

In this method the series is to be truncated offer the fourth term. Accuracy is of the order of $(\Delta t)^5$. The procedure is adopted almost same as that adopted for the second order approximation.

Let

$$t_1 = t_0 + \Delta t$$

$$x = x_0 + a_1 R_1 + a_2 R_2 + a_3 R_3 + a_4 R_4$$

$$R_1 = f(x_n\ y_n)\Delta t$$

$$R_2 = f(x_n + b_1 R_1,\ t_0 + b_2 \Delta t)\Delta t$$

$$R_3 = f(x_n + b_3 R_2,\ t_0 + b_4 \Delta t)\Delta t$$

$$R_4 = f(x_n + b_5 R_3,\ t_0 + b_6 \Delta t)\Delta t$$

$$x_{n+1} = x_n + \frac{1}{6}(R_1 + 2R_2 + 2R_3 + R_4) \qquad \text{... (8.88)}$$

where

$$a_1 = \frac{1}{6}\quad a_2 = \frac{1}{3},\quad a_3 = \frac{1}{3},\quad a_4 = \frac{1}{6}$$

$$b_1 = \frac{1}{2} = b_2 = b_3 = b_4\ ;\ b_5 = b_6 = 1$$

Δx is the incremental value of 'x' given by the weighted average of estimates based on slopes at the beginning, midpoint and at the end of the time step.

$$\frac{dx}{dt} = f(x, t)$$

x_0 and t_0 are the initial values.

Let us select very small increment to t say Δt.

Now $\quad x_1 = x_0 + \Delta x$ (given value)

$t_1 = t_0 + \Delta t$ (to be evaluated) expressing $x_1 = x_0 + \Delta x = x_0 \frac{R_1 + R_2}{2}$ ... (8.89)

where $R_1 = f(x_0, t_0)\Delta t$

$R_2 = f(x_0 + k_1, t_0 + \Delta t)\Delta t$

A general formula giving the value of "x" for the $(n+1)^{st}$ step is

$$x_{n+1} = x_n + \frac{R_1 + R_2}{2} \quad \text{... (8.90)}$$

where $R_1 = f(x_n, t_n)\Delta t$

$R_2 = f(x_n + R_1\ t_n + \Delta t)\Delta t$

8.24.6 Computational Algorithm using R-K Method

Flowchart

(See flowchart on next page.)

8.24.7 Algorithm

1. Read inertia constant, machine transient reactance, tie line reactance, voltages at generators buses, bus powers, system frequency, type of fault, duration of fault time increment and total time.
2. Conduct load flow analysis.
3. Calculate the machine currents.
4. Calaculate voltage behind transient reactance E' given by

$$E' = V_t + j x'_d I_t$$

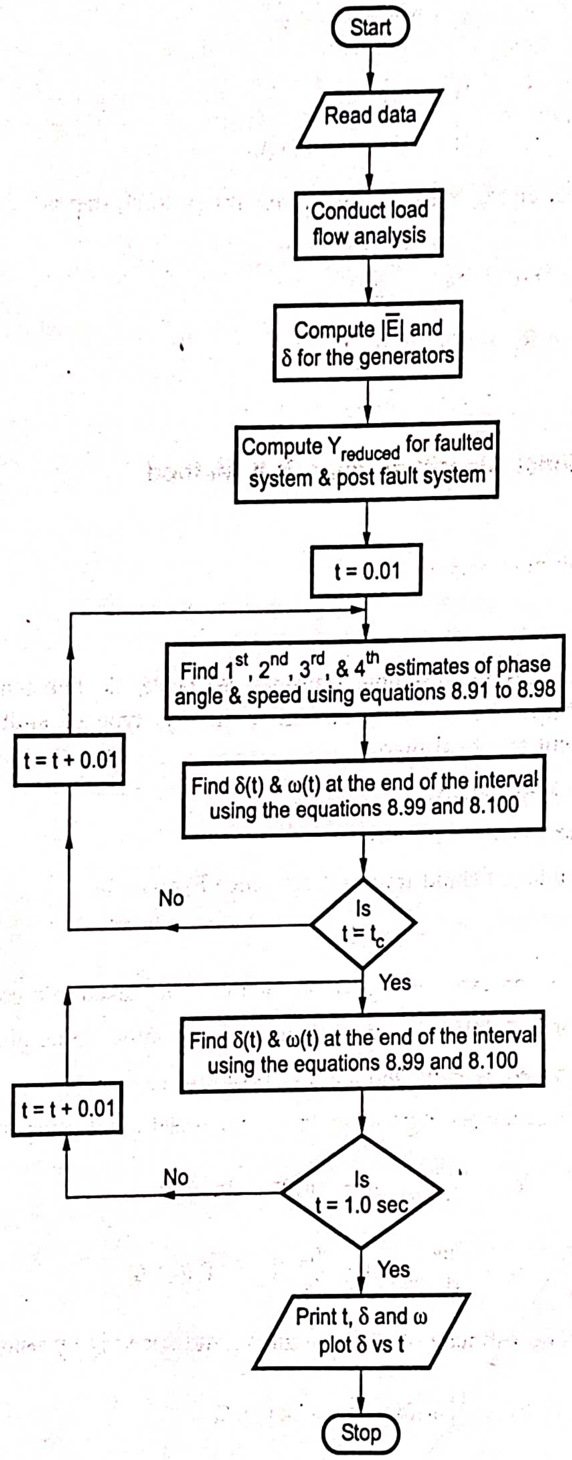
5. Assume Pm = constant since governor action is not taken into consideration.
6. The angle for E' is taken as initial value of δ and initial value of $\omega = 2\pi f_0$.
7. Compute reduced Y_{BUS} for faulted and post faulted systems.
8. Find the first estimates of phse angle k_1 and speed l_1 by using the formulae.

$$k_1 = \frac{d\delta}{dt}\Big|_{(o)} \Delta t = (\omega(0) - 2\pi f_o)\Delta t \quad \text{... (8.91)}$$

$$l_1 = \frac{d\omega}{dt}\Big|_{(o)} \Delta t = \frac{\pi f_o}{H}(P_m - P_e(0))\Delta t \quad \text{... (8.92)}$$

9. Find the second estimate of phase angle k_2 and speed l_2 by using the formulae.

$$k_2 = \left(\left(\omega(0) + \frac{l_1}{2}\right) - 2\pi f_o\right)\Delta t \quad \text{... (8.93)}$$



Transient stability analysis - computation of swing curves using R-K algorithm

$$l_2 = \frac{\pi f_o}{H}(P_m - P_{e(0)}^{(1)})\,\Delta t \quad \ldots (8.94)$$

10. Find the third estimates of phase angle k_3 and speed l_3 by using the formulae

$$k_3 = \left((\omega(0)) + \frac{l_2}{2}\right) - 2\pi f_o)\,\Delta t \quad (8.95)$$

$$l_3 = \frac{\pi f_o}{H}(P_m - P_{e(0)}^{(2)})\,\Delta t \quad \ldots (8.96)$$

11. Calculate the fourth estimates of phase angles k_4 and speed l_4 by using the formulae.

$$k_4 = \left((\omega(0)) + \frac{l_3}{2}\right) - 2\pi f_o)\,\Delta t \quad (8.97)$$

$$l_4 = \frac{\pi f_o}{H}(P_m - P_{e(0)}^{(3)})\,\Delta t \quad \ldots (8.98)$$

12. Find the phase angle and speed at the end of the first time interval

$$\delta(t) = \delta(t-1) + 1/6\,(k_1 + 2k_2 + 2k_3 + k_4) \quad \ldots (8.99)$$

$$\omega(t) = \omega(t-1) + 1/6\,(l_1 + 2\,l_2 + 2l_3 + l_4) \quad \ldots (8.100)$$

13. Repeat from step - 8 and find the phase angle and speed during the subsequent time interval till t = 0.3 sec. All these calculations should be done usin reduced Y_{BUS} matrix corresponding to faulted system.
14. Repeat the steps 8 to 12 for time t = 0.3 sec. upto t = 1.0 sec. by using post fault reduced Y_{BUS} matrix.
15. Plot swing curves and state whether the system will be stable or not.

8.24.8 Step by Step Procedure - Modified Euler Method

Step 1 : Assume initial solution $\delta^{(0)}$ and $\omega^{(0)} = \omega_s$, Δt (time step) set time curve $t_0 = 0$ and $t = t_0$.

Step 2 : Compute $\left.\frac{d\delta}{dt}\right|_{t=t}$ and $\left.\frac{d\omega}{dt}\right|_{t=t_0}$

Step 3 : Compute firm estimate of δ and ω

$$\delta^{(t+\Delta t)} = \delta^{(t)} + \Delta t \left.\frac{d\delta}{dt}\right|_{t=t_0}$$

$$\omega^{(t+\Delta t)} = \omega^{(t)} + \Delta t \left.\frac{d\omega}{dt}\right|_{t=t_0}$$

Step 4 : Compute $\left.\frac{d\delta}{dt}\right|_{t=(t_0+\Delta t)}$ and $\left.\frac{d\omega}{dt}\right|_{t=(t_0+\Delta t)}$

$$P_m = 0.5$$

$$\delta_{max} = \pi - \sin^{-1}\left[\frac{P_m}{P_{max\,3}}\right] = \pi - \sin^{-1}\left[\frac{0.5}{0.75}\right]\frac{P_{max\,1}}{P_{max\,1}} = 2.412 \text{ rad}$$

Critical clearing angle δ_c is given by

$$\cos\delta_{cr} = \frac{P_m(\delta_{max}-\delta_0) + P_{max\,3}\cos\delta_{max} - P_{max\,2}\cos\delta_0}{P_{max\,3}-P_{max\,2}}$$

$$= \frac{0.5\,[2.412-0.524] + 0.75\times\cos 2.412 - 0.2\times\cos 0.524}{0.75-0.2}$$

$$\cos\delta_{cr} = 0.385$$

$$\delta_{cr} = \cos^{-1} 0.385 = 1.176 \text{ rad} = 67.35°$$

10.9. DETERMINATION OF CRITICAL CLEARING TIME BY TRIAL AND ERROR METHOD

Critical clearing time is the maximum allowable time between the occurrence of a fault and clearing of the fault for which the system will be stable. For a given load condition and specified fault, the critical clearing time for a system is found out by trial and error method.

Assume fault clearing time for the stable operation. Then increasing the clearing time in steps till instability occurs. The value of clearing time just before instability obtained is called as the critical clearing time.

Critical time margin = Critical clearing time – Clearing time specified

$= t_{cr\,(critical)} - t_{spec}$

where t_{spec} = Specified clearing time

10.10. MODIFIED EULER METHOD

The methods used to solve swing equations are –

- Modified Euler Method
- Runge-Kutta Method

Algorithm for numerical solution of swing equation using modified Euler Method.

Numerical integration techniques can be applied to obtain approximate solutions of non-linear differential equations. Euler's method is the simplest method.

Consider a generator connected to an infinite bus through two parallel lines and a 3ϕ fault occurs at the middle of line 2 as shown in Fig.10.39.

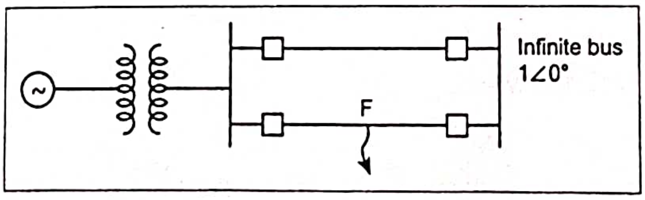


Fig. 10.39.

Let P_m be the input power which is a constant.

Prefault condition : Under steady state operation,

Power transfer from generator to an infinite bus,

$$P_e = P_m$$

$$\frac{E' V}{X_I} \sin \delta_0 = P_{max\,1} \sin \delta_0 = P_m$$

$$\sin \delta_0 = \frac{P_m}{P_{max\,1}} \Rightarrow \delta_0 = \sin^{-1}\left[\frac{P_m}{P_{max\,1}}\right] \quad \text{... (10.41)}$$

where $P_{max\,1} = \frac{E' V}{X_I}$

X_I = Transfer reactance for the prefault condition.

The rotor is running at synchronous speed,

$$\omega_0 = 2\pi f$$

Change in angular velocity is zero.

i.e., $\Delta\omega_0 = 0$

During the fault : Consider a 3ϕ fault occurs at the middle of one line 2 as shown in Fig.

$$P_{e2} = \frac{|E'||V|}{X_{II}} \sin \delta_1 = P_{max\,2} \sin \delta$$

where $P_{max\,2} = \frac{|E'||V|}{X_{II}}$

where X_{II} = Transfer reactance during the fault. (X_{II} increases during the fault).

The swing equation is given by,

$$\frac{d^2\delta}{dt^2} = \frac{\pi f}{H} [P_m - P_{max\,2} \sin \delta] = \frac{\pi f}{H} P_a$$

The above equations are transformed into the state variable form,

$$\frac{d\delta^{(1)}}{dt} = \Delta\omega \quad \text{... (10.42)}$$

$$\frac{d^2\delta}{dt^2} = \frac{d\Delta\omega^{(1)}}{dt} = \frac{\pi f P_a}{H} \quad \text{... (10.43)}$$

Compute the first estimate at $t_1 = t_0 + \Delta t$.

$$\delta^P_{i+1} = \delta_i + \left.\frac{d\delta^{(1)}}{dt}\right|_{\Delta\omega_i} \cdot \Delta t \quad \text{... (10.44)}$$

$$\Delta\omega^P_{i+1} = \Delta\omega_i + \left.\frac{d\Delta\omega^{(1)}}{dt}\right|_{\delta_i} \Delta t \quad \text{... (10.45)}$$

Compute the derivatives : Using the predicted values δ_{i+1}^{P} and $\Delta\omega_{i+1}^{P}$, determine the derivatives at the end of iteration.

$$\left.\frac{d\delta^{(2)}}{dt}\right|_{\Delta\omega_{i+1}^{P}} = \Delta\omega_{i+1}^{P} \qquad \text{... (10.46)}$$

$$\left.\frac{d\Delta\omega^{(2)}}{dt}\right|_{\delta_{i+1}^{P}} = \frac{\pi f}{H} P_a \Big|_{\delta_{i+1}^{P}} \qquad \text{... (10.47)}$$

Compute the average derivatives

$$\frac{d\delta}{dt_{ave}} = \frac{\left.\frac{d\delta^{(1)}}{dt}\right|_{\Delta\omega_i} + \left.\frac{d\delta^{(2)}}{dt}\right|_{\Delta\omega_{i+1}^{P}}}{2} \qquad \text{... (10.48)}$$

$$\frac{d\Delta\omega}{dt_{ave}} = \frac{\left.\frac{d\Delta\omega^{(1)}}{dt}\right|_{\delta_i} + \left.\frac{d\Delta\omega^{(2)}}{dt}\right|_{\delta_{i+1}^{P}}}{2} \qquad \text{... (10.49)}$$

Compute the final estimate (corrected value),

$$\delta_{i+1}^{C} = \delta_i + \left[\frac{\left.\frac{d\delta}{dt}\right|_{\Delta\omega_i} + \left.\frac{d\delta}{dt}\right|_{\Delta\omega_{i+1}^{P}}}{2}\right]\Delta t \qquad \text{... (10.50)}$$

$$\Delta\omega_{i+1}^{C} = \Delta\omega_i + \left[\frac{\left.\frac{d\Delta\omega}{dt}\right|_{\delta_i} + \left.\frac{d\Delta\omega}{dt}\right|_{\delta_{i+1}^{P}}}{2}\right]\Delta t \qquad \text{... (10.51)}$$

10.11. RUNGE-KUTTA METHOD

The following steps are involved in Runge-Kutta method to determine stability.

I estimates :

$$K_1 = \left.\frac{d\delta}{dt}\right|_{\Delta\omega_i} \times \Delta t = \Delta\omega_i \times \Delta t \qquad \text{... (10.52)}$$

$$l_1 = \left.\frac{d\Delta\omega}{dt}\right|_{\delta i} \times \Delta t = \frac{\pi f}{H}[P_m' - P_{e(\delta_i)})] \times \Delta t \qquad \text{... (10.53)}$$

II estimates :

$$K_2 = \left[\Delta\omega_i + \frac{l_1}{2}\right]\Delta t \qquad \text{... (10.54)}$$

$$l_2 = \frac{\pi f}{H}[P_m' - P_{e(\delta_i + (K_1/2))}] \times \Delta t \qquad \text{... (10.55)}$$

III estimates :

$$K_3 = \left(\Delta\omega_i + \frac{l_2}{2}\right) \times \Delta t \qquad \text{... (10.56)}$$

$$l_3 = \frac{\pi f}{H}[P_m' - P_{e(\delta_i + (K_2/2))}] \times \Delta t \qquad \text{... (10.57)}$$

Flow Chart for Modified Euler

Start

↓

Read number of buses, bus data, line data, inertia constant H, time t, change in time Δt, real power P, Power factor, etc.

↓

Assume
- Synchronous frequency voltages and currents are considered.
- Synchronous components are used in unbalanced fault representation.
- Generator voltages are unaffected by machine speed variations.

↓

Compute line current $I = \dfrac{P - jQ}{E_t^*}$

$E' = E_t + jX_d I = E' \angle \beta$

$V = E_t - j(X_{transformer} + X_{line}) \cdot I = |V| \angle \gamma$

$\delta = \beta - \gamma$

Initial bus voltage $E' = |E'| \angle \delta$,

$V = |V| \angle 0°$

↓

Compute prefault reactance X_I

$P_{e1} = \dfrac{|E'||V|}{X_I} \sin \delta_0 \Rightarrow$ find δ_0 (initial value of δ)

Assume 3ϕ fault, $\therefore P_{e2} = 0$, otherwise

$P_{e2} = \dfrac{|E'||V|}{X_{II}} \sin \delta$ during the fault

After the fault is cleared at $t = t + \Delta t$, instead of P_{e2}, substitute

$P_{e3} = \dfrac{|E'||V|}{X_{III}} \sin \delta$

↓

$\Delta t = 0.1$ sec

$\omega_0 = 2\pi f_0 = 314.519$

↓

State variable form

$$\frac{d\delta^{(1)}}{dt} = \Delta\omega$$

$$\frac{d^2\delta}{dt^2} = \frac{d\Delta\omega^{(1)}}{dt} = \frac{\pi f P_a}{H}$$

Compute the first estimate $t_1 = t_0 + \Delta t$,

$$\delta_{i+1}^{P} = \delta_i + \left.\frac{d\delta^{(1)}}{dt}\right|_{\Delta\omega_i} . \Delta t$$

$$\Delta\omega_{i+1}^{P} = \Delta\omega_i + \left.\frac{d\Delta\omega^{(1)}}{dt}\right|_{\delta_i} . \Delta t$$

Compute the derivatives: Using the predicted values δ_{i+1}^{P} and $\Delta\omega_{i+1}^{P}$, determine the derivatives at the end of iteration

$$\left.\frac{d\delta^{(1)}}{dt}\right|_{\Delta\omega_{i+1}^{P}} = \Delta\omega_{i+1}^{P}$$

$$\left.\frac{d\Delta\omega^{(2)}}{dt}\right|_{\delta_{i+1}^{P}} = \frac{\pi f}{H}\left. P_a \right|_{\delta_{i+1}^{P}}$$

Compute the average derivatives

$$\frac{d\delta}{dt_{ave}} = \frac{\left.\frac{d\delta^{(1)}}{dt}\right|_{\Delta\omega_i} + \left.\frac{d\delta^{(2)}}{dt}\right|_{\Delta\omega_{i+1}^{P}}}{2}$$

$$\frac{d\Delta\omega}{dt_{ave}} = \frac{\left.\frac{d\Delta\omega^{(1)}}{dt}\right|_{\delta_i} + \left.\frac{d\Delta\omega^{(2)}}{dt}\right|_{\delta_{i+1}^{P}}}{2}$$

Compute the final estimate (corrected value),

$$\delta_{i+1}^{C} = \delta_i + \left[\frac{\left.\frac{d\delta}{dt}\right|_{\Delta\omega_i} + \left.\frac{d\delta}{dt}\right|_{\Delta\omega_{i+1}^{P}}}{2}\right]\Delta t$$

$$\Delta\omega_{i+1}^{C} = \Delta\omega_i + \left[\frac{\left.\frac{d\Delta\omega}{dt}\right|_{\delta_i} + \left.\frac{d\Delta\omega}{dt}\right|_{\delta_{i+1}^{P}}}{2}\right]\Delta t$$

Is $\delta_{i+1} - \delta_i = 0$, $\Delta\omega_{i+1} - \Delta\omega_i = 0$?

No → iter = iter + 1; iter = 0

Yes → Print critical clearing angle δ and the corresponding critical clearing time t

End

UNIT-I

1. Per Unit Value $= \frac{\text{Actual Value}}{\text{Base Value}}$

2. Per Unit impedance $= \frac{\text{Actual impedance } (\Omega)}{\text{Base impedance } (\Omega)}$

3. Base impedance, $Z_b = \frac{KV_b^2}{MVA_b}$.

4. Base current, $I_b = \frac{KVA_b}{KV_b}$ (single-ϕ)

5. Base current, $I_b = \frac{KVA_b}{\sqrt{3} \times KV_b}$ (3-ϕ)

6. $Z_{p.u,new} = Z_{p.u,old} \times \left[\frac{KV_{b,old}}{KV_{b,new}}\right]^2 \times \left[\frac{MVA_{b,new}}{MVA_{b,old}}\right]$

(or)

$X_{pu,new} = X_{pu,old} \times \left[\frac{KV_{b,old}}{KV_{b,new}}\right]^2 \left[\frac{MVA_{b,new}}{MVA_{b,old}}\right]$

7. KV_b on LT section = KV_b on HT section $\times \frac{\text{LT Voltage rating}}{\text{HT Voltage rating}}$.

8. KV_b on HT section = KV_b on LT section $\times \frac{\text{HT Voltage rating}}{\text{LT Voltage rating}}$.

9. when specified reactance is in ohm

per unit reactance $= \frac{\text{Actual reactance } (\Omega)}{\text{Base impedance}}$

10. $Y_{bus} V = I$; Y_{bus} : bus admittance matrix.

11. $Y_{bus} = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1n} \\ Y_{21} & Y_{22} & \cdots & Y_{2n} \\ \vdots & & & \\ Y_{n1} & Y_{n2} & \cdots & Y_{nn} \end{bmatrix}$

Y_{11}, Y_{12} = Self admittances

Y_{jj} = Sum of all admittances connected to bus-j

$Y_{12}, Y_{13} \dots$ = Mutual admittances

Y_{jk} = Negative sum of all admittances connected b/w bus j & k.

Node (or) Bus Elimination:

12. $$Y_{jk,new} = Y_{jk} - \frac{Y_{jn}Y_{nk}}{Y_{nn}} \quad ; \; j = 1,2,3 \dots (n-1), \; K = 1,2,3 \dots (n-1)$$

13. Bus Impedance Matrix:

$$V = Z_{bus} I.$$

14. Adding Z_b from a new bus -p to existing bus-q. (case ii) (just copy)

$$Z_{bus,new} = \left[Z_{exg} \right.$$

5. Adding Z_b from an existing bus-q to reference bus. Case (i

i) case (ii) perform operation.

ii) Node Elimination.

$$Z_{jk,act} = Z_{jk} - \frac{Z_{j(n+1)} Z_{(n+1)k}}{Z_{(n+1)(n+1)}}$$

$$\left[Z_{12} = Z_{21} \; ; \; Z_{13} = Z_{31} \; \text{etc} \dots \right].$$

16. Adding Z_b between two existing buses p and q.

$$Z_{bus,new} = \left[\begin{array}{c|c} Z_{orig} & \begin{matrix} Z_{1p} - Z_{1q} \\ Z_{2p} - Z_{2q} \\ \vdots \\ Z_{np} - Z_{nq} \end{matrix} \\ \hline Z_{p1}-Z_{q1} \quad Z_{p2}-Z_{q2} \cdots Z_{pn}-Z_{qn} & Z_{(n+1)(n+1)} \end{array}\right]$$

$$Z_{(n+1)(n+1)} = Z_b + Z_{pp} + Z_{qq} - 2Z_{pq}.$$

then perform node elimination.

17. Direct determination of a bus impedance matrix :

$$Z_{bus} = [Z_b]$$

UNIT - II

Gauss - Seidal Method :

$$Q_{p,cal}^{k+1} = (-1)Im\left\{ (V_p^k)^* \left[\sum_{q=1}^{p-1} Y_{pq} V_q^{k+1} + \sum_{q=p}^{n} Y_{pq} V_q^k \right] \right\}$$

$$Q_{p,cal}^{k+1} < Q_{p,min} \quad \text{then } Q_p = Q_{p,min}.$$

$$Q_{p,cal}^{k+1} > Q_{p,max}, \text{ then}, Q_p = Q_{p,max}.$$

$$V_{p,temp}^{k+1} = \frac{1}{Y_{pp}} \left[\frac{P_p - jQ_p}{(V_p^k)^*} - \sum_{q=1}^{p-1} Y_{pq} V_q^{k+1} - \sum_{q=p+1}^{n} Y_{pq} V_q^k \right].$$

3. $\delta_p^{k+1} = \tan^{-1}\left[\frac{\text{Imag. part of } V_{p,\text{temp}}^{k+1}}{\text{Real part of } V_{p,\text{temp}}^{k+1}}\right]$

4. $V_p^{k+1} = |V_p|_{spec} \angle \delta_p^{k+1}$.

5. $V_p^{k+1} = \frac{1}{Y_{pp}}\left[\frac{P_p - jQ_p}{(V_p^k)^*} - \sum_{q=1}^{p-1} Y_{pq} V_q^{k+1} - \sum_{q=p+1}^{n} Y_{pq} V_q^k\right]$

6. $V_{p,acc}^{k+1} = V_p^k + \alpha(V_p^{k+1} - V_p^k)$

7. $\Delta V_p^{k+1} = V_p^{k+1} - V_p^k$.

Newton-Raphson Method [Polar-Form]

1. $P_p^{k+1} = |V_p^k| \sum_{q=1}^{n} |Y_{pq}||V_q^k| \cos(\theta_{pq} + \delta_q^k - \delta_p^k)$

2. $Q_p^{k+1} = -|V_p^k| \sum_{q=1}^{n} |Y_{pq}||V_q^k| \sin(\theta_{pq} + \delta_q^k - \delta_p^k)$

3. $\Delta P_p^{k+1} = P_{p,spec}^k - P_p^{k+1}$.

4. $\Delta Q_p^{k+1} = Q_{p,spec}^k - Q_p^{k+1}$.

5. $Q_p^{k+1} < Q_{p,min} \Rightarrow Q_p^{k+1} = Q_{p,min}$.

6. $Q_p^{k+1} > Q_{p,max} \Rightarrow Q_p^{k+1} = Q_{p,max}$.

7. Elements of J_A :

If $(p \neq q)$ $= \dfrac{\partial P_p}{\partial \delta_q} = -|V_p^k||Y_{pq}||V_q^k| \sin(\theta_{pq} + \delta_q^k - \delta_p^k)$

If $(p = q)$ $= \dfrac{\partial P_p}{\partial \delta_p} = \sum_{\substack{q=1 \\ \neq p}}^{n} |V_p||V_q||Y_{pq}| \sin(\theta_{pq} + \delta_q - \delta_p)$

8. Elements of J_B :

If $(p \neq q)$ $\cdot \dfrac{\partial P_p}{\partial |V_q|} = |V_p||Y_{pq}| \cos(\theta_{pq} + \delta_q - \delta_p)$

If $(p = q)$ $\dfrac{\partial P_p}{\partial V_p} = 2|V_p||Y_{pp}| \cos(\theta_{pp}) + \sum_{\substack{q=1 \\ \neq p}}^{n} |V_q||Y_{pq}| \cos(\theta_{pq} + \delta_q - \delta_p)$

9. Elements of J_C :

If $(p \neq q)$ $= \dfrac{\partial Q_p}{\partial \delta_q} = -|V_p||V_q||Y_{pq}| \cos(\theta_{pq} + \delta_q - \delta_p)$

If $(p = q)$ $= \dfrac{\partial Q_p}{\partial \delta_p} = \sum_{\substack{q=1 \\ \neq p}}^{n} |V_p||V_q||Y_{pq}| \cos(\theta_{pq} + \delta_q - \delta_p)$.

10. Elements of J_D :

If $(p \neq q)$ $= \dfrac{\partial Q_p}{\partial |V_q|} = -|V_p||Y_{pq}| \sin(\theta_{pq} + \delta_q - \delta_p)$

If $(p = q)$ $\cdot \dfrac{\partial Q_p}{\partial V_p} = -2|V_p||Y_{pp}| \sin\theta_{pp} - \sum_{\substack{q=1 \\ \neq p}}^{n} |V_q||Y_{pq}| \sin(\theta_{pq} + \delta_q - \delta_p)$

11. $$\begin{bmatrix} \Delta \delta \\ \Delta V \end{bmatrix} = \begin{bmatrix} J_A & J_B \\ J_C & J_D \end{bmatrix}^{-1} \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}$$

12. $|V_p^{k+1}| = |V_p^k| + |\Delta V_p^k|$

13. $\delta_p^{k+1} = \delta_p^k + \Delta \delta_p^k$

14. $P_{p,spec} = P_G - P_L$.

15. $Q_{p,spec} = Q_G - Q_L$

Fast decoupled Power Flow Method.

1. $P_p^{k+1} = |V_p^k| \sum_{q=1}^{n} |Y_{pq}||V_q^k| \cos(\theta_{pq} + \delta_q^k - \delta_p^k)$

2. $Q_p^{k+1} = -|V_p^k| \sum_{q=1}^{n} |Y_{pq}||V_q^k| \sin(\theta_{pq} + \delta_q^k - \delta_p^k)$

3. $\Delta P_p^{k+1} = P_{p,spec}^{k} - P_{p,cal}^{k+1}$

4. $\Delta Q_p^{k+1} = Q_{p,spec}^{k} - Q_{p,cal}^{k+1}$

5. $Q_p^{k+1} > Q_{p,max}$, then $Q_p^{k+1} = Q_{p,max}$.

6. $Q_p^{k+1} < Q_{p,min}$, then $Q_p^{k+1} = Q_{p,min}$.

7. $[\Delta \delta_p] = -[B']^{-1} \left[\frac{\Delta P_p}{|V_p|}\right]$

8. $[\Delta V_p] = -[B'']^{-1} \left[\frac{\Delta Q_p}{|V_p|}\right]$.

9. $|V_p^{k+1}| = |V_p^k| + |\Delta V_p^{k+1}|$

10. $\delta_p^{k+1} = \delta_p^k + \Delta \delta_p^{k+1}$.

1. Subtransient reactance:

$$X_d'' = X_l + \frac{1}{\left(\frac{1}{X_a} + \frac{1}{X_f} + \frac{1}{X_{dw}}\right)}$$

2. Transient Reactance:

$$X_d' = X_l + \frac{1}{\left(\frac{1}{X_a} + \frac{1}{X_f}\right)}$$

3. Steady State Reactance:

$$X_d = x_l + x_a.$$

4. p.u value = $\frac{\text{Actual Value}}{\text{Base Value}}$

5. Base Current $I_b = \frac{KVA_b}{\sqrt{3} \times KV_b}$ (or) $\frac{MVA_b}{\sqrt{3} \times kV_b}$

6. $I_f'' = I_g'' + I_m''$

7. $I_f = \frac{V_{th}}{Z_{th}}$ (or) $I_f = \frac{V_{th}}{Z_{th} + Z_f}$ (if Z_f given)

8. Fault current of Motor, $I_m'' = -I_L + \Delta I_m$

9. Fault current of generator, $I_g'' = I_L + \Delta I_g$.

10. Rated Momentary current = 1.6 × Symmetrical subtransient fault current.

11. Short circuit Interrupting current = Multiplying Factor × Transient Short Circuit current.

Speed of CB	Multiplying Factor
8 cycles or more	1
5 cycles	1.1
3 cycles	1.2
2 cycles	1.4
1½ cycles	1.5

12. Impedances Connected in parallel.

$$\frac{1}{Z_{th}} = \frac{1}{Z_1} + \frac{1}{Z_2} + \cdots$$

13. Short circuit capacity (SCC)

$$|SCC| = |V^{\circ}||I_f|$$

14. $$|SCC|_{3\phi} = \frac{S_{b,3\phi}}{|Z_{th}|\ p.u}$$

15. $$I_f = \frac{V_q^{\circ}}{Z_{qq} + Z_f}.$$

16. $V_q^f = V_q^{\circ} - Z_{qk} I_f$; k : fault bus no.

17. $$\begin{bmatrix} \Delta V_1 \\ \vdots \\ \Delta V_q \\ \vdots \\ \Delta V_N \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1q} & \cdots & Z_{1N} \\ \vdots & \vdots & & & & \\ Z_{q1} & Z_{q2} & \cdots & Z_{qq} & \cdots & Z_{qN} \\ \vdots & \vdots & & & & \\ Z_{N1} & Z_{N2} & \cdots & Z_{Nq} & \cdots & Z_{NN} \end{bmatrix} \begin{bmatrix} 0 \\ \vdots \\ -I_q \\ \vdots \\ 0 \end{bmatrix}.$$

18. $I_{pq}^{f} = \frac{V_p^f - V_q^f}{Z_{pq}}$.

19. $\left[V_{bus}^{o}\right] = \begin{bmatrix} V_1^o \\ \vdots \\ V_q^o \\ \vdots \\ V_N^o \end{bmatrix}$

UNIT-IV

1. $1+a+a^2=0$
2. $a^3=1$
3. $a=-0.5+j0.866$
4. $a^2=-0.5-j0.866$
5. $V_a = V_{a0}+V_{a1}+V_{a2}$
6. $V_b = V_{b0}+V_{b1}+V_{b2}$
7. $V_c = V_{c0}+V_{c1}+V_{c2}$
8. $V_{b0} = V_{a0} = V_{c0}$
9. $V_{b1} = a^2 V_{a1}$; $V_{c1} = a V_{a1}$
10. $V_{b2} = a V_{a2}$; $V_{c2} = a^2 V_{a2}$.
11. $\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}$ unbalanced Vectors (Voltage)
12. $\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$ Unbalanced Current Vectors.
13. $\begin{bmatrix} V_{a0} \\ V_{b0} \\ V_{c0} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$ Symmetrical Voltage Components.

14. $$\begin{bmatrix} I_{a0} \\ I_{b0} \\ I_{c0} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$ Symmetrical current Components.

15. $Z_0 = 3Z_n + Z_{g0}$

16. $V_{a1} = E_a - I_{a1}Z_1$

17. $V_{a2} = -I_{a2}Z_2$

18. $V_{a0} = -I_{a0}Z_0$

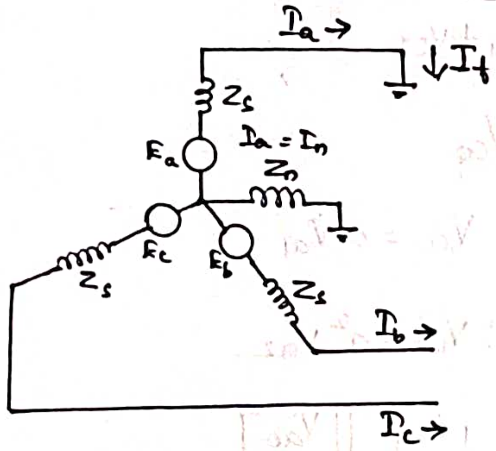
19. $$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix}\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}.$$

20. Single line to ground fault: on an unloaded Generator

$I_b = 0$

$I_c = 0$

$V_a = 0$



$I_{a0} = I_{a1} = I_{a2} = \frac{I_a}{3}$

$$I_{a1} = \frac{E_a}{Z_0 + Z_1 + Z_2}$$

$I_f = I_a = 3I_{a1}$

21. Line - Line fault on an unloaded Generator:

$V_b = V_c$

$V_{a1} = V_{a2}$

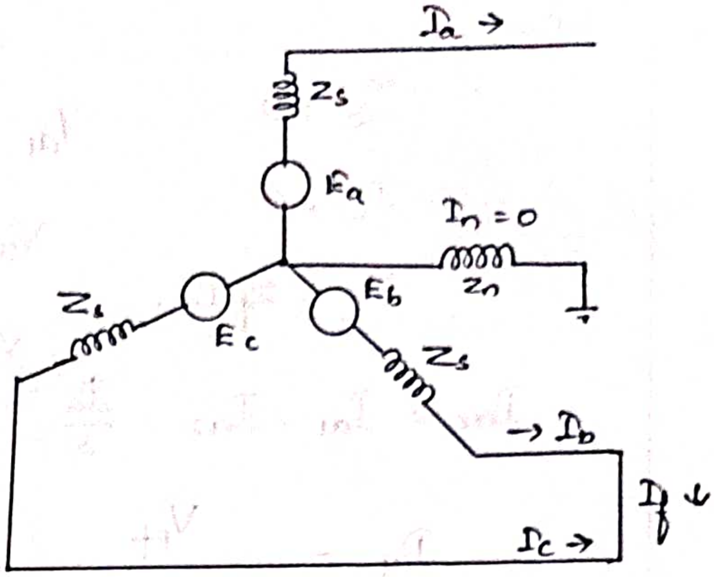
$I_a = 0$

$I_b + I_c = 0$

$I_b = -I_c$

$I_{a2} = -I_{a1}$

$I_{a1} = \frac{E_a}{Z_1 + Z_2}$



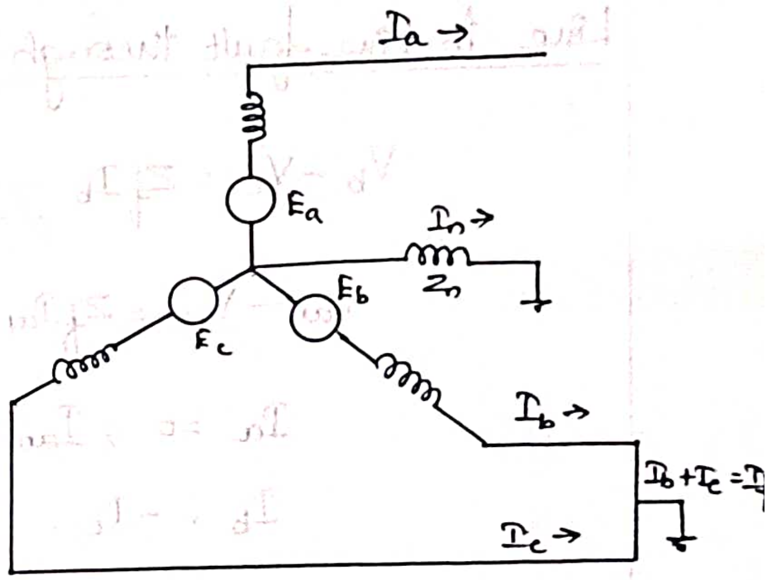
$I_f = I_b = I_{a1}(a^2 - a)$

<u>Double Line to Ground Fault : on an unloaded Generator.</u>

$V_b = 0$

$V_c = 0$

$V_{a0} = V_{a1} = V_{a2} = \frac{V_a}{3}$



$I_a = 0$

$I_{a0} = -\frac{E_a}{Z_0} + \frac{I_{a1} Z_1}{Z_0} = \frac{-V_{a0}}{Z_0}$

$I_{a1} = I_{a1}$

$I_{a2} = -\frac{E_a}{Z_2} + \frac{I_{a1} Z_1}{Z_2} = \frac{-V_{a2}}{Z_2}$

$I_{a1} = \frac{E_a}{Z_1 + \frac{Z_0 Z_2}{Z_0 + Z_2}}$

$I_f = 2I_{a0} + (a + a^2)(I_{a1} + I_{a2}) = I_b + I_c$

Single Line to Ground Fault through Impedance (Z_f)

$I_b = 0$

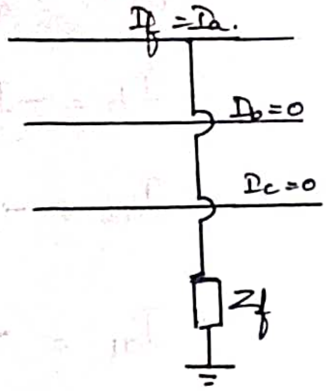
$I_c = 0$

$V_a = Z_f I_a$.

$V_{a1} = -Z_0 I_{a1}$

$V_{a1} = E_a - Z_1 I_{a1}$

$V_{a2} = -Z_2 I_{a1}$



$I_{a0} = I_{a1} = I_{a2} = \frac{I_a}{3}$

$$I_{a1} = \frac{V_{pf}}{Z_0 + Z_1 + Z_2 + 3Z_f}$$

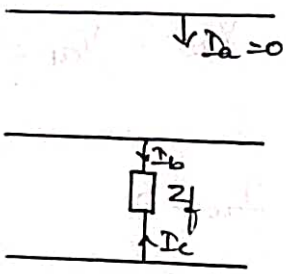
V_{pf} : Prefault Voltage at fault

$I_f = 3I_{a1}$

Line to Line fault through Impedance:

$V_b - V_c = Z_f I_b$

$V_{a1} - V_{a2} = Z_f I_{a1}$



$I_a = 0$; $I_{a0} = 0$

$I_b = -I_c$.

$I_{a2} = -I_{a1}$

$$I_{a1} = \frac{V_{pf}}{Z_1 + Z_2 + Z_f}$$

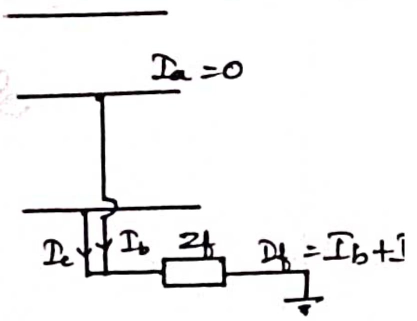
$V_{a0} = 0$; $V_{a1} = V_{a2}$

$I_f = I_b = -j\sqrt{3} I_{a1}$

Double Line to Ground Fault through Impedance (Z_f)

$V_b = V_c = Z_f (I_b + I_c)$.

$$V_{a0} - V_{a1} = V_b = Z_f (I_b + I_c)$$



$$I_a = 0$$

$$I_b + I_c = 3I_{a0}$$

$$I_{a1} = \frac{V_{pf}}{Z_1 + \frac{Z_2 (Z_0 + 3Z_f)}{Z_2 + Z_0 + 3Z_f}}$$

$$I_f = I_b + I_c = 3I_{a0}.$$

$$V_{a0} = V_{a2} + 3Z_f I_{a0}.$$

$$I_{a2} = \frac{-I_{a1}(Z_0 + 3Z_f)}{Z_1 + Z_0 + 3Z_f}$$

$$I_{a0} = \frac{-I_{a1} \times Z_0}{Z_1 + Z_0 + 3Z_f}.$$

UNIT - V

1. Swing Equation.

$$\frac{H}{\pi f} \frac{d^2\delta}{dt^2} = P_m - P_e$$

2. $P_e = P_{max} \sin \delta$.

3. Equal Area Criterion

$$\frac{d\delta}{dt} = 0, \text{ the term } \int_{\delta_0}^{\delta} P_a . d\delta = 0$$

4. $$\delta_{2,max} = \pi - \sin^{-1}\left(\frac{P_{m1,max}}{P_{max}}\right)$$

5. Critical Clearing Angle

$$\delta_{cc} = \cos^{-1}\left[\frac{P_m}{P_{max}}(\delta_{max} - \delta_0) + \cos\delta_{max}\right]$$

6. Critical Clearing Time

$$t_{cc} = \sqrt{\frac{2H\ \delta_{cc} - \delta_0}{\pi f\ P_m}}$$